Dr. Philippe J.S. De Brouwer

# INTRODUCTION TO QUANTUM COMPUTING

XII KNMF CONFERENCE

ALL RIGHTS RESERVED BY THE AUTHOR

April 4, 2024

# Table of Contents

# Contents

| 1.2       Quantum Computers       1         2       Basics of Quantum Physics       6         2.1       Quantum states and superposition       2         2.2       Quantum interference       2         3       Quantum Bits (Qubits)       10         3.1       Understanding Qubits       11         4       Quantum Gates and Circuits       11         4.1       Differences between classical bits and qubits       12         4.2       Introduction to quantum gates       12         4.3       Measuring Qubits       14         4.4       Building quantum circuits       14         4.5       What are quantum computers really       14         5       Quantum Algorithms       16         5.2       Examples: Shor's algorithm       17         5.4       Note: Ciphering       17         5.4       Note: Ciphering       18         5.5       Solving Sparse Large Linear Systems       18         6.1       Models of Quantum Computing       19         6.2       Physical Realisations of Qubits       19         6.3       Quantum Supremacy       22         6.4       Current state of quantum hardware       22  | 1 | Inti | roduction                           | 3         |
|--|---|------|-------------------------------------|-----------|
| 2       Basics of Quantum Physics       6         2.1       Quantum states and superposition       5         2.2       Quantum entanglement       6         2.3       Quantum interference       6         3       Quantum Bits (Qubits)       10         3.1       Understanding Qubits       10         4       Quantum Gates and Circuits       11         4.1       Differences between classical bits and qubits       11         4.2       Introduction to quantum gates       12         4.3       Measuring Qubits       14         4.4       Building quantum circuits       14         4.5       What are quantum computers really       14         5       Quantum Algorithms       16         5.1       Overview of quantum algorithms       16         5.2       Examples: Grover's algorithm       16         5.4       Note: Ciphering       17         5.4       Note: Ciphering       18         5.5       Solving Sparse Large Linear Systems       18         6       How to build a quantum computer       19         6.1       Models of Quantum Computing       19         6.2       Physical Realisations of Qubits       19  |   | 1.1  | Turing Machines                     | 4         |
| 2.1Quantum states and superposition52.2Quantum entanglement82.3Quantum interference93Quantum Bits (Qubits)103.1Understanding Qubits104Quantum Gates and Circuits114.1Differences between classical bits and qubits114.2Introduction to quantum gates124.3Measuring Qubits144.4Building quantum circuits144.5What are quantum computers really145Quantum Algorithms165.1Overview of quantum algorithms165.2Examples: Grover's algorithm165.3Examples: Grover's algorithm165.4Note: Ciphering185.5Solving Sparse Large Linear Systems186How to build a quantum Computing196.2Physical Realisations of Qubits196.3Quantum Supremacy206.4Current state of quantum hardware207Challenges in Quantum Computing227.1Decoherence and error correction227.2Scalability issues228Future of Quantum Computing228.1The Road-map248.2Potential applications24   |   | 1.2  | Quantum Computers                   | 5         |
| 2.2       Quantum entanglement       8         2.3       Quantum interference       9         3       Quantum Bits (Qubits)       10         3.1       Understanding Qubits       10         4       Quantum Gates and Circuits       11         4.1       Differences between classical bits and qubits       12         4.2       Introduction to quantum gates       12         4.3       Measuring Qubits       14         4.4       Building quantum circuits       14         4.5       What are quantum computers really       14         5       Quantum Algorithms       16         5.1       Overview of quantum algorithms       16         5.2       Examples: Shor's algorithm       16         5.3       Examples: Grover's algorithm       17         5.4       Note: Ciphering       18         5.5       Solving Sparse Large Linear Systems       18         6       How to build a quantum computer       19         6.1       Models of Quantum Computing       19         6.2       Physical Realisations of Qubits       19         6.3       Quantum Supremacy       20         7       Challenges in Quantum Computing       21  | 2 | Bas  | ics of Quantum Physics              | 6         |
| 2.3       Quantum interference       9         3       Quantum Bits (Qubits)       10         3.1       Understanding Qubits       10         4       Quantum Gates and Circuits       11         4.1       Differences between classical bits and qubits       11         4.2       Introduction to quantum gates       12         4.3       Measuring Qubits       14         4.4       Building quantum circuits       14         4.5       What are quantum computers really       16         5.1       Overview of quantum algorithms       16         5.2       Examples: Shor's algorithm       16         5.3       Examples: Grover's algorithm       17         5.4       Note: Ciphering       18         5.5       Solving Sparse Large Linear Systems       18         6       How to build a quantum computer       19         6.1       Models of Quantum Computing       19         6.2       Physical Realisations of Qubits       19         6.3       Quantum Supremacy       22         6.4       Current state of quantum hardware       22         7.1       Decoherence and error correction       23         7.2       Scalability issues  |   |      |                                     | 7         |
| 3       Quantum Bits (Qubits)       10         3.1       Understanding Qubits       10         4       Quantum Gates and Circuits       11         4.1       Differences between classical bits and qubits       12         4.2       Introduction to quantum gates       12         4.3       Measuring Qubits       12         4.4       Building quantum circuits       14         4.5       What are quantum computers really       14         5       Quantum Algorithms       16         5.1       Overview of quantum algorithms       16         5.2       Examples: Shor's algorithm       16         5.3       Examples: Grover's algorithm       16         5.4       Note: Ciphering       18         5.5       Solving Sparse Large Linear Systems       18         6       How to build a quantum computer       19         6.1       Models of Quantum Computing       19         6.2       Physical Realisations of Qubits       19         6.3       Quantum Supremacy       20         6.4       Current state of quantum hardware       20         7       Challenges in Quantum Computing       21         7.1       Decoherence and error correct   |   |      |                                     | 8         |
| 3.1       Understanding Qubits       10         4       Quantum Gates and Circuits       11         4.1       Differences between classical bits and qubits       12         4.2       Introduction to quantum gates       12         4.3       Measuring Qubits       14         4.4       Building quantum circuits       14         4.5       What are quantum computers really       14         4.5       What are quantum algorithms       16         5.1       Overview of quantum algorithms       16         5.2       Examples: Shor's algorithm       16         5.3       Examples: Grover's algorithm       17         5.4       Note: Ciphering       18         5.5       Solving Sparse Large Linear Systems       18         6       How to build a quantum computer       19         6.1       Models of Quantum Computing       19         6.2       Physical Realisations of Qubits       19         6.3       Quantum Supremacy       20         7       Challenges in Quantum Computing       21         7.1       Decoherence and error correction       22         7.2       Scalability issues       24         8       Future of Quantum Comp   |   | 2.3  | Quantum interference                | 9         |
| 4       Quantum Gates and Circuits       11         4.1       Differences between classical bits and qubits       12         4.2       Introduction to quantum gates       12         4.3       Measuring Qubits       14         4.4       Building quantum circuits       14         4.5       What are quantum computers really       14         4.5       What are quantum computers really       14         5       Quantum Algorithms       16         5.1       Overview of quantum algorithms       16         5.2       Examples: Shor's algorithm       16         5.3       Examples: Grover's algorithm       17         5.4       Note: Ciphering       18         5.5       Solving Sparse Large Linear Systems       18         6       How to build a quantum computer       19         6.1       Models of Quantum Computing       19         6.2       Physical Realisations of Qubits       19         6.3       Quantum Supremacy       20         7       Challenges in Quantum Computing       21         7.1       Decoherence and error correction       22         7.2       Scalability issues       24         8       Future of Quantum Co   | 3 | Qua  |                                     | 10        |
| 4.1       Differences between classical bits and qubits       11         4.2       Introduction to quantum gates       12         4.3       Measuring Qubits       14         4.4       Building quantum circuits       14         4.5       What are quantum computers really       14         5       Quantum Algorithms       16         5.1       Overview of quantum algorithms       16         5.2       Examples: Shor's algorithm       16         5.3       Examples: Grover's algorithm       16         5.4       Note: Ciphering       17         5.4       Note: Ciphering       18         5.5       Solving Sparse Large Linear Systems       18         6       How to build a quantum computer       19         6.1       Models of Quantum Computing       19         6.2       Physical Realisations of Qubits       19         6.3       Quantum Supremacy       20         6.4       Current state of quantum hardware       20         7       Challenges in Quantum Computing       21         7.1       Decoherence and error correction       22         7.2       Scalability issues       24         8       Future of Quantum Computing </td <td></td> <td>3.1</td> <td>Understanding Qubits</td> <td>10</td> |   | 3.1  | Understanding Qubits                | 10        |
| 4.2Introduction to quantum gates144.3Measuring Qubits144.4Building quantum circuits144.5What are quantum computers really145Quantum Algorithms165.1Overview of quantum algorithms165.2Examples: Shor's algorithm165.3Examples: Grover's algorithm165.4Note: Ciphering175.4Note: Ciphering185.5Solving Sparse Large Linear Systems186How to build a quantum computer196.1Models of Quantum Computing196.2Physical Realisations of Qubits196.3Quantum Supremacy206.4Current state of quantum hardware207Challenges in Quantum Computing217.1Decoherence and error correction217.2Scalability issues248Future of Quantum Computing248.1The Road-map248.2Potential applications24  | 4 | Qua  | antum Gates and Circuits            | 11        |
| 4.3 Measuring Qubits144.4 Building quantum circuits144.5 What are quantum computers really15 <b>5 Quantum Algorithms</b> 165.1 Overview of quantum algorithms165.2 Examples: Shor's algorithm165.3 Examples: Grover's algorithm165.4 Note: Ciphering175.5 Solving Sparse Large Linear Systems186 How to build a quantum computer196.1 Models of Quantum Computing196.2 Physical Realisations of Qubits196.3 Quantum Supremacy206.4 Current state of quantum hardware207 Challenges in Quantum Computing217.1 Decoherence and error correction217.2 Scalability issues248 Future of Quantum Computing248.1 The Road-map248.2 Potential applications24   |   | 4.1  |                                     | 11        |
| 4.4       Building quantum circuits       14         4.5       What are quantum computers really       13         5       Quantum Algorithms       16         5.1       Overview of quantum algorithms       16         5.2       Examples: Shor's algorithm       16         5.3       Examples: Grover's algorithm       16         5.4       Note: Ciphering       17         5.4       Note: Ciphering       18         5.5       Solving Sparse Large Linear Systems       18         6       How to build a quantum computer       19         6.1       Models of Quantum Computing       19         6.2       Physical Realisations of Qubits       19         6.3       Quantum Supremacy       20         6.4       Current state of quantum hardware       20         7       Challenges in Quantum Computing       21         7.1       Decoherence and error correction       22         7.2       Scalability issues       24         8       Future of Quantum Computing       24         8.1       The Road-map       24         8.2       Potential applications       24  |   |      |                                     | 12        |
| 4.5       What are quantum computers really       14         5       Quantum Algorithms       16         5.1       Overview of quantum algorithms       16         5.2       Examples: Shor's algorithm       16         5.2       Examples: Grover's algorithm       16         5.3       Examples: Grover's algorithm       17         5.4       Note: Ciphering       18         5.5       Solving Sparse Large Linear Systems       18         6       How to build a quantum computer       19         6.1       Models of Quantum Computing       19         6.2       Physical Realisations of Qubits       19         6.3       Quantum Supremacy       20         6.4       Current state of quantum hardware       20         7       Challenges in Quantum Computing       22         7.1       Decoherence and error correction       22         8       Future of Quantum Computing       24         8.1       The Road-map       24         8.2       Potential applications       24  |   | 4.3  |                                     | 14        |
| 5       Quantum Algorithms       16         5.1       Overview of quantum algorithms       16         5.2       Examples: Shor's algorithm       16         5.3       Examples: Grover's algorithm       17         5.4       Note: Ciphering       17         5.5       Solving Sparse Large Linear Systems       18         5.5       Solving Sparse Large Linear Systems       18         6       How to build a quantum computer       19         6.1       Models of Quantum Computing       19         6.2       Physical Realisations of Qubits       19         6.3       Quantum Supremacy       20         6.4       Current state of quantum hardware       20         7       Challenges in Quantum Computing       21         7.1       Decoherence and error correction       22         7.2       Scalability issues       24         8       Future of Quantum Computing       24         8.1       The Road-map       24         8.2       Potential applications       24  |   |      |                                     | 14        |
| 5.1       Overview of quantum algorithms       16         5.2       Examples: Shor's algorithm       16         5.3       Examples: Grover's algorithm       17         5.4       Note: Ciphering       17         5.5       Solving Sparse Large Linear Systems       18         5.5       Solving Sparse Large Linear Systems       18         6       How to build a quantum computer       19         6.1       Models of Quantum Computing       19         6.2       Physical Realisations of Qubits       19         6.3       Quantum Supremacy       20         6.4       Current state of quantum hardware       20         7       Challenges in Quantum Computing       21         7.1       Decoherence and error correction       22         8       Future of Quantum Computing       24         8.1       The Road-map       24         8.2       Potential applications       24  |   | 4.5  | What are quantum computers really   | 15        |
| 5.2       Examples: Shor's algorithm       16         5.3       Examples: Grover's algorithm       17         5.4       Note: Ciphering       18         5.5       Solving Sparse Large Linear Systems       18         6       How to build a quantum computer       19         6.1       Models of Quantum Computing       19         6.2       Physical Realisations of Qubits       19         6.3       Quantum Supremacy       20         6.4       Current state of quantum hardware       20         7       Challenges in Quantum Computing       21         7.1       Decoherence and error correction       22         8       Future of Quantum Computing       24         8.1       The Road-map       24         8.2       Potential applications       24   | 5 | Qua  |                                     | 16        |
| 5.3       Examples: Grover's algorithm       17         5.4       Note: Ciphering       18         5.5       Solving Sparse Large Linear Systems       18         6       How to build a quantum computer       19         6.1       Models of Quantum Computing       19         6.2       Physical Realisations of Qubits       19         6.3       Quantum Supremacy       20         6.4       Current state of quantum hardware       20         7       Challenges in Quantum Computing       21         7.1       Decoherence and error correction       21         7.2       Scalability issues       24         8       Future of Quantum Computing       24         8.1       The Road-map       24         8.2       Potential applications       24   |   | 5.1  |                                     | 16        |
| 5.4       Note: Ciphering  |   |      |                                     | 16        |
| 5.5       Solving Sparse Large Linear Systems       18         6       How to build a quantum computer       19         6.1       Models of Quantum Computing       19         6.2       Physical Realisations of Qubits       19         6.3       Quantum Supremacy       20         6.4       Current state of quantum hardware       20         7       Challenges in Quantum Computing       22         7.1       Decoherence and error correction       22         7.2       Scalability issues       24         8       Future of Quantum Computing       24         8.1       The Road-map       24         8.2       Potential applications       24  |   | 5.3  |                                     | 17        |
| 6 How to build a quantum computer       19         6.1 Models of Quantum Computing   |   | -    |                                     | 18        |
| 6.1 Models of Quantum Computing       19         6.2 Physical Realisations of Qubits       19         6.3 Quantum Supremacy       20         6.4 Current state of quantum hardware       20         7 Challenges in Quantum Computing       21         7.1 Decoherence and error correction       21         7.2 Scalability issues       24         8 Future of Quantum Computing       24         8.1 The Road-map       24         8.2 Potential applications       24  |   | 5.5  | Solving Sparse Large Linear Systems | 18        |
| 6.2 Physical Realisations of Qubits       19         6.3 Quantum Supremacy       20         6.4 Current state of quantum hardware       20         7 Challenges in Quantum Computing       21         7.1 Decoherence and error correction       21         7.2 Scalability issues       22         8 Future of Quantum Computing       24         8.1 The Road-map       24         8.2 Potential applications       24   | 6 |      | w to build a quantum computer       | 19        |
| 6.3       Quantum Supremacy       20         6.4       Current state of quantum hardware       20         7       Challenges in Quantum Computing       21         7.1       Decoherence and error correction       21         7.2       Scalability issues       24         8       Future of Quantum Computing       24         8.1       The Road-map       24         8.2       Potential applications       24  |   |      |                                     | 19        |
| 6.4       Current state of quantum hardware       20         7       Challenges in Quantum Computing       23         7.1       Decoherence and error correction       23         7.2       Scalability issues       24         8       Future of Quantum Computing       24         8.1       The Road-map       24         8.2       Potential applications       24   |   |      |                                     | 19        |
| 7       Challenges in Quantum Computing       23         7.1       Decoherence and error correction       24         7.2       Scalability issues       24         8       Future of Quantum Computing       24         8.1       The Road-map       24         8.2       Potential applications       24  |   |      |                                     | 20        |
| 7.1       Decoherence and error correction       23         7.2       Scalability issues       24         8       Future of Quantum Computing       24         8.1       The Road-map       24         8.2       Potential applications       24   |   | 6.4  | Current state of quantum hardware   | 20        |
| 7.2       Scalability issues       24         8       Future of Quantum Computing       24         8.1       The Road-map       24         8.2       Potential applications       24   | 7 | Cha  |                                     | 23        |
| 8       Future of Quantum Computing       24         8.1       The Road-map       24         8.2       Potential applications       24   |   |      |                                     | 23        |
| 8.1       The Road-map       24         8.2       Potential applications       24  |   | 7.2  | Scalability issues                  | 24        |
| 8.2 Potential applications $\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 2^4$   | 8 | Fut  |                                     | <b>24</b> |
| 11   |   |      |                                     | 24        |
| 8.3 Case Study: HSBC   |   |      | **                                  | 24        |
|  |   | 8.3  | Case Study: HSBC                    | 27        |

| 9 | Lim | its of Quantum Computers  | 29 |
|---|-----|---------------------------|----|
|   | 9.1 | Problem Complexity limits | 29 |
|   | 9.2 | Limits in Applications    | 30 |
|   |     |                           |    |

Introduction

**10 Conclusions** 

1

The three stages of computers: (1) Analogue



44,000 BCE



500 BCE



Egypt/China, Na

Napier's bones, John Napier 1617 Charle



 $\mathbf{31}$ 

Charles Babage's difference engine, 1820s

Figure 1: Analogue counting devices

The Analytical Engine – 1837 (concept)

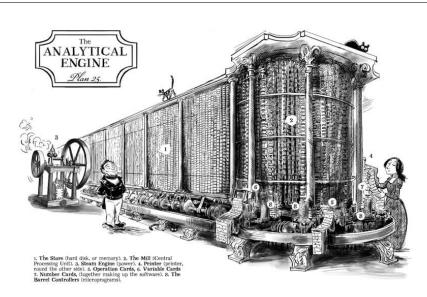


Figure 2: In 1837 Charles Babage proposed the first general purpose computer: the "analytical engine". Legend: 1: memory, 2: the mill (CPU), 3: steam engine, 4: printer, 5: operation cards, 6: variable cards, 7: number cards, 8: barrel (controller)

# 1.1 Turing Machines

The three stages of computers: (2) Digital



Alan Turing, 1937 – bomba 1939



Eniac, 1947



First PC, IBM 1981



Supercomputer

Figure 3: Digital Computers

The three stages of computers: (2) Digital

$$-4 -$$

Introduction to Quantum Computing



Figure 4: The fastest supercomputer in the world: Frontier, HPE CRAY EX235A, AMD OPTIMIZED 3RD GENERATION EPYC 64C 2GHZ – USA, Oakr Ridge – Rmax = 1.5 Exa Flops =  $1.5 \times 10^{18}$  Flops, using 21'000KwH – foto: Oak Ridge

# 1.2 Quantum Computers

The three stages of computers: (3) Quantum

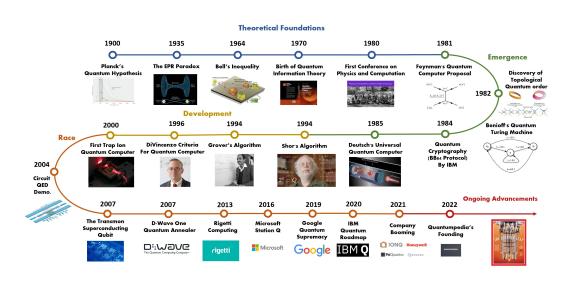


Figure 5: The timeline for quantum computers

# 2 Basics of Quantum Physics

# The quantum world

Imagine a world where

- things are largely empty space (much more than 99.99999999999996% empty)
- things are waves and waves are things
- things can be in an infinite amount of places at the same time
- it is not possible to observe anything without changing what we observe forever and everywhere
  - so and event on one planet can influence reality in another galaxy, and
  - this influencing happens faster than the speed of light
- it is possible to get through walls even without sufficient energy to do so
- where no properties like color, softness, compassion, intelligence, cold, wet, etc. exists
- things have only mathematical properties
- vacuum is not empty

Could this world underlie our familiar and logical world?

# 2.1 Quantum states and superposition

Thomas Young's double slit experiment (1801)

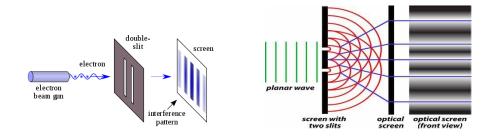


Figure 6: The double slit experiment. — (images licensed under Creative Commons CC0 1.0 Universal Public Domain Dedication and Creative Commons Attribution-Share Alike 3.0 Unported (author Fu-Kwun Hwang))

### Schrödinger's Equation

Quantum entities are described by the Schödinger equation:

$$i\hbar\frac{\partial}{\partial t}\Psi({\bf r},t)=\hat{H}\Psi({\bf r},t)$$

The probabilities to find the entity are then given by

$$P(\mathbf{r},t) = |\Psi(\mathbf{r},t)|^2$$

### Superposition

The equation is linear, hence linear combinations of solutions are also solutions.

Example: Qubit

If an object can have a quantum state "up" or "down" with equal probabilities, then it is described by  $\Psi = \frac{1}{\sqrt{2}} |up\rangle + \frac{1}{\sqrt{2}} |down\rangle$ . When measured one state is observed.

### Schrödinger's Cat thought experiment

-7-

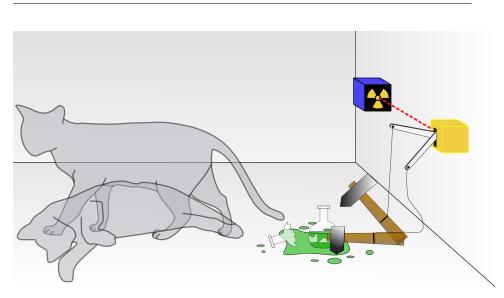


Figure 7: Poison is released when the radioactive atom decayes. As long as the box is not opened the radioactive atom is in superposition  $\Psi_{atom} = \alpha_1 |decayed\rangle + \alpha_2 |not decayed\rangle$ , and hence the cat must be  $\Psi_{cat} = \alpha_1 |dead\rangle + \alpha_2 |alive\rangle$ .

# 2.2 Quantum entanglement

### Entanglement

A system of two qubits can be characterized by

 $\alpha_1 |00\rangle + \alpha_2 |01\rangle + \alpha_3 |10\rangle + \alpha_4 |11\rangle$ 

where

- $|01\rangle$  means: the first qubit is  $|0\rangle$  and the sec- Figure 8: AI's interpretaond  $|1\rangle$
- $\sum_{i=1}^{4} |\alpha_i|^2 = 1$ , with  $\forall i : \alpha_i \in \mathbb{C}$

### Entanglement

If two or more of  $\alpha_i$  are non-zero, qubits are entangled if knowing one determines the state of the other.

Example  $\frac{\sqrt{2}}{2} |11\rangle + \frac{\sqrt{2}}{2} |10\rangle \text{ is not entangled} \\ \frac{\sqrt{2}}{2} |01\rangle + \frac{\sqrt{2}}{2} |10\rangle \text{ is entangled}$ 



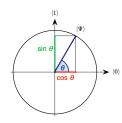
tion of wedding rings in entanglement. Microsoft's copilot



Figure 9: AI's interpretation of entanglement. Mi-

#### $\mathbf{2.3}$ Quantum interference

#### **Amplitudes and Probabilities**



For a single qubit: unit sphere in  $\mathbb{C}^2$  with the quantum state  $\alpha_1|0\rangle + \alpha_2|1\rangle$  such that  $|\alpha_1|^2 + |\alpha_2|^2 = 1$ . Notes

- The state can be re-written as  $|\cos \theta|^2 + |\sin \theta|^2 = 1$ , or  $|\alpha_1|^2 = \cos^2 \theta$  and  $|\alpha_2|^2 = \sin^2 \theta$ .
- $|\alpha_1|^2$  is the probability of measuring  $|0\rangle$  and  $|\alpha_2|^2$  is the probability of measuring  $|1\rangle$ .

### Amplitudes are Complex

Probabilities are real numbers and add up to 1, amplitudes are complex and the sum of absolute values adds up to 1. This allows for wave-like behaviour: interference.

### **Quantum Interference**

-9-

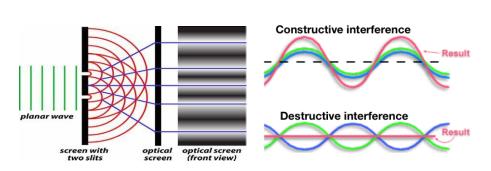


Figure 10: Quantum particles can influence others or themselves (via superposition) and disappear in certain places.

Well ...

Is the universe local and real?



Figure 11: AI's interpretation of a universe that is not local nor real. Microsoft's copilot

# 3 Quantum Bits (Qubits)

# 3.1 Understanding Qubits

# The QuBit

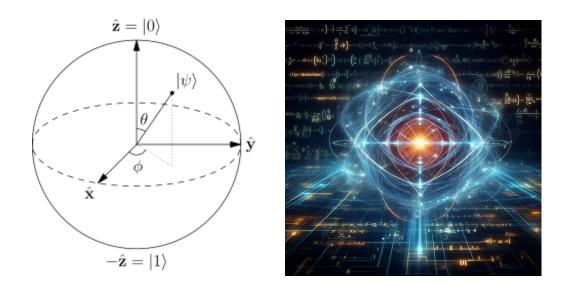


Figure 12: The qubit can be visualized on the Figure 13: AI's interpretation Bloch-Sphere. Image licensed under Creative of a qubit. Microsoft's copilot Commons

# 4 Quantum Gates and Circuits

# 4.1 Differences between classical bits and qubits

**Classical Computers** 

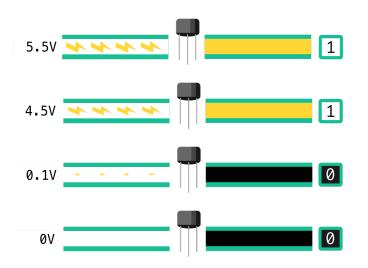


Figure 14: We use transistors to create logical states of 1 and 0.

# Logical Gates

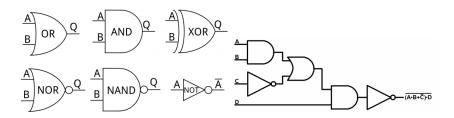


Figure 15: Those transistors are used to create logical gates that are in turn building blocks for logical circuits.

# 4.2 Introduction to quantum gates

# Quantum Gates

### quantum gate

a quantum logic gate (or quantum gate) is a basic quantum circuit operating on a small number of qubits.

#### **Examples of Quantum Gates**

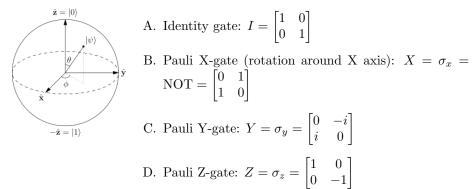
| Operator                         | Gate(s)                           |          | Matrix   |
|----------------------------------|-----------------------------------|----------|--|
| Pauli-X (X)                      | - <b>x</b> -                      |          | $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$   |
| Pauli-Y (Y)                      | - <b>Y</b> -                      |          | $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$  |
| Pauli-Z (Z)                      | $-\mathbf{z}$                     |          | $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$  |
| Hadamard (H)                     | - <b>H</b> -                      |          | $rac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1\\ 1 & -1 \end{bmatrix}$   |
| Phase (S, P)                     | - <b>S</b> -                      |          | $\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$   |
| $\pi/8$ (T)                      | - <b>T</b> -                      |          | $\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$  |
| Controlled Not<br>(CNOT, CX)     | $\stackrel{\bullet}{\rightarrow}$ |          | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$   |
| Controlled Z (CZ)                | - <b>Z</b> -                      | <b>1</b> | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$  |
| SWAP                             |                                   |          | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$   |
| Toffoli<br>(CCNOT,<br>CCX, TOFF) |                                   |          | $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ |

Figure 16: Examples of popular quantum gates. There are in fact an uncountable infinity of quantum gates.

### Examples of quantum gates on one qubit

The vector representation of  $|a\rangle = \alpha_1 |1\rangle + \alpha_2 |0\rangle$  is  $\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$ 

Examples acting on one qubit:



### Example of quantum gates: creating superposition

**Hadamard Gate** acts on a single qubit. It maps the basis states  $|0\rangle \mapsto \frac{|0\rangle+|1\rangle}{\sqrt{2}}$  and  $|1\rangle \mapsto \frac{|0\rangle-|1\rangle}{\sqrt{2}}$  (an equal superposition state if given a computational basis state).

The two states  $(|0\rangle + |1\rangle)/\sqrt{2}$  and  $(|0\rangle - |1\rangle)/\sqrt{2}$  are sometimes written  $|+\rangle$  and  $|-\rangle$  respectively. The Hadamard gate performs a rotation of  $\pi$  about the axis  $(\hat{x} + \hat{z})/\sqrt{2}$  at the Bloch sphere, and is therefore involutory.

-13 -

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

If the Hermitian  $(H^{\dagger} = H^{-1} = H)$  Hadamard gate is used to perform a change of basis, it flips  $\hat{x}$  and  $\hat{z}$ . For example, HZH = X and  $H\sqrt{X} H = \sqrt{Z} = S$ .

#### Example of a quantum gate on 2 qubits and entanglement

Controlled gates act on 2 or more qubits, where one or more qubits act as a control for some operation.

### controlled NOT gate (or CNOT or CX)

acts on 2 qubits, and performs the NOT operation on the second qubit only when the first qubit is  $|1\rangle$  (otherwise leaves it unchanged). With respect to the basis  $|00\rangle$ ,  $|01\rangle$ ,  $|10\rangle$ ,  $|11\rangle$  it is represented by the Hermitian unitary matrix:

$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

### 4.3 Measuring Qubits

#### **Measuring Qubits**

Measurement = reduce the quantum states to a classical state. Therefore, measurement is irreversible and not a quantum gate.

#### The probability of finding a state is the modulus of its amplitude<sup>1</sup>

if 
$$\Psi = \alpha |x\rangle + \dots$$
, then  $P[|x\rangle] = |\alpha|^2$ 

For example, measuring a qubit with the quantum state  $\frac{|0\rangle - i|1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1\\ -i \end{bmatrix}$  will yield with equal probability either  $|0\rangle$  or  $|1\rangle$ 

# 4.4 Building quantum circuits

Building your first quantum circuit



See the presentation of

later today ;-)

(c) Philippe De Brouwer

-14 -

 $<sup>^1{\</sup>rm This}$  is known as the Born rule and appears as a stochastic non-reversible operation as it sets with a given probability the quantum state equal to the basis vector that represents the measured state.

#### 4.5What are quantum computers really

What is a quantum computer?



Figure 17: Photosynthesis is possible thanks to quantum mechanics. - own photo 2014



### An example of a simulation: the Fermiac

Figure 18: The FERMIAC, or Monte Carlo trolley, was an analog device invented by Enrico Fermi to implement studies of neutron transport. — image under Creative Commons Attribution-Share Alike 1.0

# Aspects of Quantum Computing: Exponential Power

• qubit  $\rightarrow 2$  quantum states dimensions:  $\alpha |0\rangle + \beta |1\rangle$ 

- 2 qubits  $\rightarrow$  4 states:  $\alpha_1 |00\rangle + \alpha_2 |01\rangle + \alpha_3 |10\rangle + \alpha_4 |11\rangle$
- 3 qubits  $\rightarrow$  8 quantum state dimensions
- 6 qubits  $\rightarrow$  64 quantum state dimensions (card deck)
- 10 qubits  $\rightarrow$  1024 quantum state dimensions (810 listed companies on WSE)
- 20 qubits  $\rightarrow 1.048576 \times 10^6$  quantum state dimensions (ca. number of all possible liquid investments)
- 60 qubits  $\rightarrow 1.1529215 \times 10^{18}$  states (ca.  $10^{19}$  grains of sand on earth)
- 175 qubits  $\rightarrow 4.7890486 \times 10^{52}$  states (ca.  $10^{50}$  atoms on earth)
- 275 qubits  $\rightarrow$  6.0708403  $\times$   $10^{82}$  quantum states (ca.  $10^{82}$  atoms in the visible universe)

### Note: entanglement

To simulate quantum states on a Turing machine, we need to encode all possible entangled states too. The number of states in a quantum processor is  $2^N$ , the complexity with entanglement scales as follows:

- A. 10 qubits  $\rightarrow$  1,024 quantum states  $\xrightarrow{\text{entanglement}}$  16,000 Bits = 16 KB
- B. 500 qubits  $\rightarrow$  more quantum states than atoms in the visible universe  $\xrightarrow{\text{entanglement}}$  not enough atoms in the visible universe

# 5 Quantum Algorithms

5.1 Overview of quantum algorithms

### 5.2 Examples: Shor's algorithm

# Factoring

PGP relies on factoring large numbers

(c) Philippe De Brouwer

-16 -

| $\# \ {f digits}$ | Supercomputer                   | Quantum comp. |
|-------------------|---------------------------------|---------------|
| 10,000            | 0 s                             | 56 s          |
| 100,000           | 0.6 year                        | $2 \min$ .    |
| 200,000           | 78,254 yrs                      | $2 \min$ .    |
| 300,000           | 449 mln. yrs                    | $2 \min$ .    |
| 400,000           | $72~\mathrm{x}$ age of universe | $3 \min$ .    |

Introduction to Quantum Computing

### Factoring

Shor's Algorythm in quantum computers does not scale exponentially

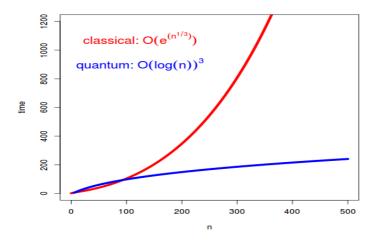


Figure 19: Time needed to factor large numbers in classical approach and with quantum computers

# 5.3 Examples: Grover's algorithm

# Programming a Universal Quantum Computer

Lov Grover's Algorithm

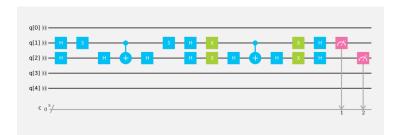
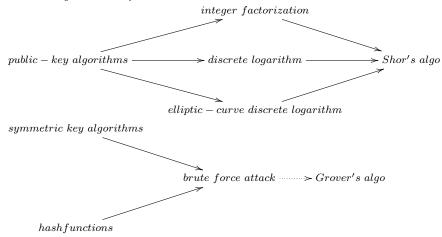


Figure 20: Grover's algorithm only needs  $O(\sqrt{N})$  steps to find matching entry in unstructured data.

# 5.4 Note: Ciphering

# Breaking Codes and Passwords

Shor's Alogorithm to factor numbers



5.5 Solving Sparse Large Linear Systems

### Large Linear Systems

| $\begin{bmatrix} A_{11} \end{bmatrix}$ | $\ldots A_{1N}$ | $\begin{bmatrix} x_1 \end{bmatrix}$ | $\begin{bmatrix} b_1 \end{bmatrix}$ | with up to $s$ non-zero |
|--|-----------------|-------------------------------------|-------------------------------------|-------------------------|
|  |                 | × : =                               |                                     | $A_{ij}$ per row/column |
| · ·                                    | •               |                                     | 1 · 1                               | and condition number    |
| $A_{M1}$                               | $\ldots A_{MN}$ | $\lfloor x_N \rfloor$               | $\lfloor b_N \rfloor$               | k                       |

Classical methods solve this in  $O(Nsk) \dots$  quantum algorithms need O(log(N)sk)

(c) Philippe De Brouwer

-18 -

# 6 How to build a quantum computer

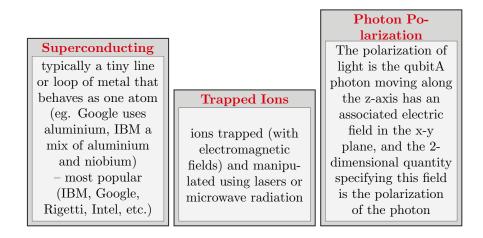
# 6.1 Models of Quantum Computing

Models of Quantum Computing

| Gate Model   | Measurement<br>Based  | Adiabatic   |
|--|---|---|
| also Circuit Model.<br>Uses entangled<br>qubits and gates to<br>execute operations | set up entangled<br>state over all<br>qubits and then<br>measure one by one | start from flat<br>energy surface and<br>build up slowly,<br>expect the system<br>to stay in the<br>lowest energy state |
|  | Annealing   |   |
|  | not mathemati-<br>cally equivalent  |   |

# 6.2 Physical Realisations of Qubits

Physical Realisations of Qubits



-19 -

| Spin Qubits           |                                    |                   |
|-----------------------|------------------------------------|-------------------|
| Elementary particles  |                                    |                   |
| and composite         |                                    | Energy Levels of  |
| particles carry an    | Topological                        | Hydrogen Atom     |
| intrinsic angular mo- |                                    |                   |
| mentum called spin.   | using Majorana                     | The electron in a |
| The quantum           | using Majorano<br>Zero-Mode Quasi- | hydrogen atom can |
| mechanical spin       | particles (sort of                 | be in its ground  |
| state of an electron  | non-Abelian anyon)                 | state or in an    |
| or proton can be      | non-Abenan anyon)                  | excited state.    |
| used as qubits        |                                    |                   |

# 6.3 Quantum Supremacy

Quantum Supremacy

**Definition 1** (quantum supremacy). Quantum supremacy is the potential ability of quantum computing devices to solve problems that classical computers practically cannot.

Expectation: 50 sufficiently coherent q-bits needed for quantum supremacy.

**Definition 2** (quantum advantage). Quantum advantage is the potential to solve problems faster. In computational complexity-theoretic terms, this generally means providing a superpolynomial speedup over the best known or possible classical algorithm.

# 6.4 Current state of quantum hardware

Current State: Quantum Supremacy overconfident claims

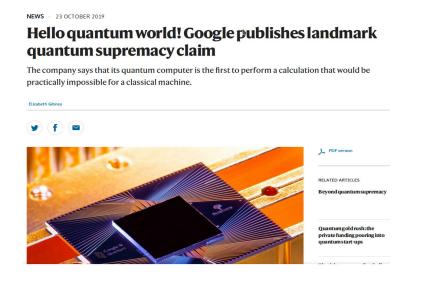


Figure 21: Submitted, October 1<sup>st</sup>, 2024 – https://arxiv.org/abs/2403.00910

# Current State: Quantum Supremacy with annealers

| () Cornell University  | We gratefully acknowledge s  | upport from the Simons Foundation, <u>memi</u><br><u>Institutions</u> , and all contributors. <u>Don</u>  |
|--|--|---|
| ∃ <b>r ∕i V</b> > quant-ph > arXiv 2403.00910  | Search<br>Help   Advan   | All fields V Search   |
| Quantum Physics<br>[Scienters 1 Mr 202]<br>Computational supremacy in quantum simulation<br>Andrew D. King, Alberto Nocera, Mark M. Ram, Jacek Dziamaga, Roeland Wersema, William Bernoudy, Jac  | sk Raymond, Nitin  | Access Paper:<br>• View PDF<br>• HTML (experimental)<br>• Tex Source<br>• Other Formats   |
| W X Mig, Audelli Nocietà, Marten M Ralles, Jacke D Zalmiaga, robental Wintserles, Vinianti Derindury, Jack X Ha<br>Jan Notas Hensdork, Richard Hans, Katll Soothy, Faho J Monare, Andrea Y. Derkiey, Martin Bochank, Kevi<br>rani, Samantha Cibere, Jake Connor, Martin H. Dehn, Rahul Deshpande, Sana Ejemaee, Pau Fanë, Kelkey Hu<br>nonald, Gaelem Marsden, Carbierne C. McGeoch, Pezza Molav, Richard Neufeld, Mann Norouzoour, Travis OL),<br>Potonas, Gabrio Puolin–Janarer, Thomas Prescot, Mauricio Felis, China Filos, Mohammad Samanai, Benjamin<br>ov, Edward Sterpica, Berta Trullas Clavera, Nicholas Tsai, Mark Volkmann, Alexander Whitcar, Jed D. Whittaker<br>son, Jason Yao, T.J. Y, Anders W, Sandvik, Gonzalo Alvarez, Roger G. Melko, Juan Carrasquille, Marcel Franz<br>sntum computers hold the pronise of solving certain problems that le beyond the reach of conventional computers. Establish<br>habilay, sepecialy for impachti and meningful problems, remains a central chalenge. Dre such problem is the simulation of<br>and rescues that givo exponentially with system size. Here we show that superconducing quantum annealing processors<br>and resources that givo exponentially with system size. Here we show that superconducing quantum annealing of entagent<br>and resources that givo exponentially with system size. Here we show that superconducing quantum annealing of censors<br>and approaches. We sees supproving the relation base franze methods and or tensors and methods and conducide quantum annealing and resources that givor exponential scaling of effi-<br>tel quench in two-, three- and intitle-dimensional giva glasses, supporting the observed stretcher-opponential scaling of effi-<br>sion of practical garbance that scaling computers changes of the stretcher convertence and therefore and therefore and computers that annealer within a reasonable infordance that disease changes and the stretcher convertication dark and the stretcher changes and t | ey Hamer, Emile<br>Li, Allison J.R.<br>Oh, Joel Pasvolsky,<br>amin Sheldan, Anatoly<br>ittaker, Warren               | Vere larere Current browse context: quant.ph < prev   next > new   recent   2403 Change to browse by. cond-mat. cond-mat.ds-rn cond-mat.stat-mech |
|  | lation of<br>ssical simulations<br>rocessors can rapidly<br>entanglement in the<br>ing of effort for<br>hat no known | References & Citations INSPIRE HEP NASAADS Google Scholar Semantic Scholar  |
|  |  | Export BibTeX Citation<br>Bookmark<br>곳 숲   |
| Subject: Quantum Physics (geant ph), Disordered Systems and Neural Networks (cond-mat.dis-m); Statistical Mechanics (cond-mat.statement)<br>(or not/2413.00010) (geant-ph)<br>(or not/2413.00010) (geant-ph) for this version)<br>respirate anglo 48550an/v2413.00010<br>Company (Statistical Mechanics)   | )  |   |

Figure 22: Submitted, March 1<sup>st</sup>, 2024 – https://arxiv.org/abs/2403.00910

# **D-Wave**



Figure 23: The quantum computer of D-Wave (pictures: D-Wave) - since 2007

### Adiabatic Algorithm



Figure 24: https://www.dwavesys.com/quantum-computing

Logical Quibits: recent progress: 2024-03-04



Figure 25: https://thequantuminsider.com 2024-04-03 – also on https://blogs.microsoft.com and https://www.quantinuum.com.

# 7 Challenges in Quantum Computing

# 7.1 Decoherence and error correction

Decoherence

### Note: temperature

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

with:

- $v_{rms}$  the average speed of a molecule in a gas in  $\frac{m}{s}$
- $k = 1.38 x 10^{-23} \frac{J}{K}$
- T the temperature in Kelvin
- m the molecular mass in Kg

# Coherence and Decoherence

Systems interacting with the environment in which they reside generally become entangled with that environment, a phenomenon known as quantum decoherence. This can explain why, in practice, quantum effects are difficult to observe in systems larger than microscopic.

-23 -

# 7.2 Scalability issues

Scalability



Figure 26: Intel Corporation's 49-qubit quantum computing test chip, "Tangle Lake," – 2018. Credit: Intel Corporation

# 8 Future of Quantum Computing

# 8.1 The Road-map

# IBM's Road-map

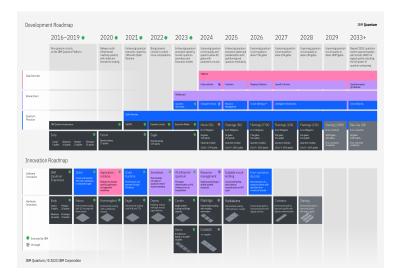


Figure 27: IBM's Quantum Roadmap (newsroom.ibm.com)

# 8.2 Potential applications

• Quantum Physics modelling: most obvious application is to understand quantum mechanical systems better

- **Biochemical modeling**: from determining the 3D shape of a protein to gene expression, the calculation of complex biological molecules to the atoms could revolutionize biotechnology research.
- **Climate modeling**: Climate models are extraordinarily complex and stretch the limits of what current supercomputers can do. A better understanding of the climate, with a finer calculation scale in the model, both geographically and in time, could help in understanding climate change risks.
- Material Science: Understanding quantum physics better and the reaction of materials down to individual atoms can open new designs for materials used in aerospace, batteries, 3D printing, manufacturing, etc.
- Semiconductors, chip design, qubits: Quantum computers could be used to make computer chips a lot more powerful. With "normal" chips now reaching the nanometre scale, quantum phenomena become increasingly problematic, and quantum computers might be needed to solve them.
- **Cryptography**: Quantum computers could potentially make all current cryptography methods obsolete. This is a serious concern for military, financial & IT systems. But at the same time, it could make cryptography even more secure.
- Optimizations: financial markets, traffic optimization, etc.



Figure 28: McKinsey Quantum Technology Monitor (April 2023) predicts USD 1.3 trillion in value by 2035 - source: https://www.mckinsey.com

-25 -

# Use cases in banking

- Optimization:
  - A. portfolio optimization
  - B. collateral optimization
  - C. stress testing
  - D. transaction settlement
  - E. asset pricing
  - F. ATM replenishment
- Machine Learning
  - fraud detection
  - credit scoring
  - synthetic data and data augmentation
- Simulations:
  - random number generator
  - Monte Carlo, LPDE simulations, etc.
  - asset valuation
  - ES and VaR calculations
- Encryption:
  - quantum key encryption
  - quantum currency
  - quantum blockchain

### **Resulting Advantages**

quadratic to exponential speedup

- better risk management
- lower costs
- greener computing
- better forecasting
- more suitable investment
- $\bullet~{\rm etc.}$

Boston Consulting Group estimates a value of 42B to 67B for financial institutions

# 8.3 Case Study: HSBC

# Why is HSBC interested

- Quantum computing could revolutionise financial services in areas like portfolio optimisation, fraud detection and cybersecurity.
- Quantum computers promise to deliver a step-change in computational power, with the potential to tackle highly complex tasks far beyond the capabilities of today's machines
- The quantum sector is estimated USD1.3 trillion in value by 2035

source: HSBC and quantum

### HSBC's strategy

- A. Working with a range of **organisations like IBM**, **Fujitsu and Quantinuum**, **leading academic institutions**, and **governmental organisations**, to put us at the forefront of the financial services industry in exploring how to integrate quantum computing into our products and services
- B. Building a **dedicated quantum research team** and in-house team of PhD scientists at HSBC to formalise our use cases into deep research projects and develop patents and quantum products
- C. **Bank-wide strategy**: Collaborating across business lines and functions to develop real world use cases to improve our processes and prepare for a quantum-secure economy

source: HSBC and quantum

**Proofs of Concept in HSBC** 

-27 -

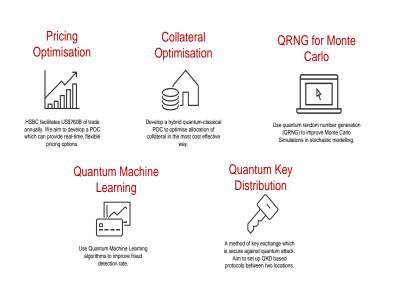


Figure 29: Proofs of concept in HSBC. source: HSBC and quantum

Quantum Key Distribution in HSBC

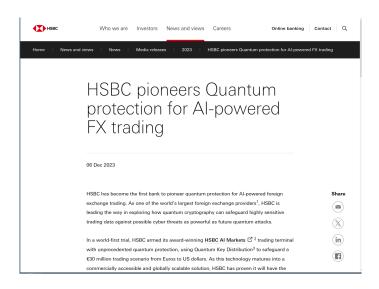


Figure 30: Proofs of concept in HSBC: quantum key distribution.  $_{\rm source:\ HSBC\ and\ quantum}$ 

### HSBC's Philip Intallura

Introduction to Quantum Computing

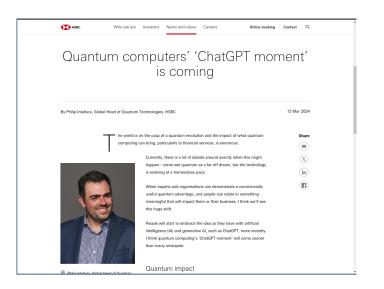


Figure 31: Proofs of concept in HSBC: quantum key distribution. Source: HSBC

# 9 Limits of Quantum Computers

# 9.1 Problem Complexity limits

Limits of Quantum Computers: Complexity Theory

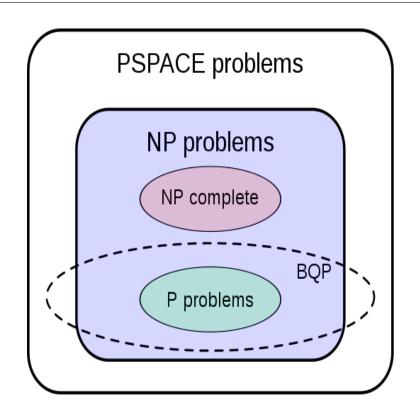


Figure 32: BQP –bounded-error quantum polynomial time– is the quantum equivalent of BPP –bounded-error probabilistic polynomial time

# 9.2 Limits in Applications

# Turing Machines are Turing Complete

### **Turing Complete**

A system is Turing complete if it can simulate any Turing machine, meaning it can compute any Turing-computable function. Essentially, it can perform any calculation that a computer with unlimited resources could. Most modern programming languages are Turing complete.

In practical terms, a Turing Complete system means a system in which a program can be written that will find an answer, although with no guarantees regarding runtime or memory use.

### Quantum Computers are impractical for many applications

While a (theoretical) Quantum Turing Machine is Turing Complete, there are much practical barriers.

# 10 Conclusions

# Conclusions: Q-Day is near

I predict that in 1 to 10 years quantum computers will bring us

- insight in quantum physics
- new medications, better batteries, better materials, etc.
- other encryption
- the ability to to gather more data and use it
- all kinds of optimizations, such as better optimized investment portfolios
- Artificial General Intelligence
- greener computing (e.g. bitcoin alone is responsible for 1.5% of the world's  $CO_2$  production)
- but most exciting: ... answers to questions that we don't know yet.

# Further Reading

- Michio Kaku, Quantum Supremacy: How the Quantum Computer Revolution Will Change Everything order on Amazon.com
- McKinsey, McKinsey Quantum Technology Monitor, April 2023 download
- McKinsey, 2020, "How quantum computing could change financial services" download
- IBM, "The Quantum Decade" (e-book) download
- E. Rieffel and W Polak, MIT Press, "Quantum Computing, a Gentle Introduction" download
- Quantum Computing for the Quantum Curious, C. Hughes et al., Springer download
- a list of books: download