THE BIG-R BOOK

FROM DATA SCIENCE TO LEARNING MACHINES AND BIG DATA

— PART 06—

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THE BIG R-BOOK:

From Data Science to Big Data and Learning Machines

♡— PART 06: Introduction to Companies —♡

These slides are to be used in with the book – for best experience, teachers will read the book *before* using the slides and students have access to the book and the code.

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chapter 28:

Financial Accounting (FA)

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section 1:

The Statements of Accounts

Definition 1 (Income Statement)

The Income Statement is all cash income minus all cash expenses.

The Profit and Loss Statement

Definition 2 (P & L)

Net sales (= revenue = sales)
- Cost of goods sold
= Gross profit
- SG&A expenses (combined costs of operating the company)
- R&D
= EBITDA
- Depreciation and amortization
= EBIT
- Interest expense (cost of borrowing money)
= EBT
- Tax expense
= Net income (EAT)

Definition 3 (NOPAT)

NOPAT = Net Operating Income After Taxes (this is EAT minus extra ordinary income)

This NOPAT is essential when we want to understand how good a company is doing on its core business, and that is important to understand how good it will be doing in the future, because extra ordinary profit or loss is unlikely to be repeated. Instead of defining NOPAT by what it does not contain, it can also be written in terms of its constituents.

$$
\begin{aligned}
NOPAT = & (\text{Net Income} - \text{after-tax Non-operating Gains} \\
& + \text{after-tax Non-operating Losses} + \text{after-tax Interest Expense}) \\
& \approx \text{Operating Profit}(1 - \text{tax rate})
\end{aligned}
$$

With: Operating Profit = EBIT - non operating income

Definition 4 (Balance Sheet)

Assets	**Liabilities and Owner's Equity**
Fixed Assets (Non-current Assets)	Shareholders Equity
	= Captial Stock + Retained Earnings
Current Assets	Current Liabilities
= Liquid Assets + Stock	

Detailed breakdown of assets

- Current Assets
  - Cash and cash equivalents.
  - Accounts receivable.
  - Prepaid expenses for future services that will be used within a year.
- Non-Current assets (Fixed Assets)
  - Property, plant and equipment.
  - Investment property, such as real estate held for investment purposes.
  - Intangible assets.
  - Financial assets (excluding investments accounted for using the equity method, accounts receivables, and cash and cash equivalents), such as notes receivables.
  - Investments accounted for using the equity method.
  - Biological assets, which are living plants or animals. Bearer biological assets are plants or animals which bear agricultural produce for harvest, such as apple trees grown to produce apples and sheep raised to produce wool.

Detailed breakdown of Liabilities

- Accounts payable
- Provisions for warranties or court decisions (contingent liabilities that are both probable and measurable)
- Financial liabilities (excluding provisions and accounts payables), such as promissory notes and corporate bonds
- Tax liabilities
- Deferred tax liabilities and deferred tax assets
- Unearned revenue for services paid for by customers but not yet provided
- Shares

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section 2:

The Value Chain

Value Creation

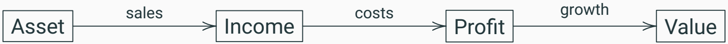

Figure 1: The elements of wealth creation in a company. The company acquires an assets, uses this to generate income, we take into account the costs that were incurred and have the profit. The potential to generate and accumulate profit constitutes the value of a company.

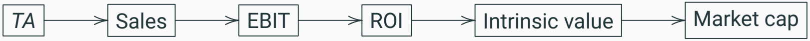

Figure 2: KPIs of the Value Chain that can be used by a manager who wants to increase the value of a company. TA stands for “total assets”, “EBIT for “earnings before interest and taxes”, and ROI is “return on investments” – we clarify these concepts further in this chapter.

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section 3:

Further Terminology

Definition 5 (Loans)

loans := debt = a sum of money borrowed with the obligation to be pay back at pre-agreed terms and conditions.

Note – Wording loans an debt

For the purpose of this section, we will refer to loans (or debt) as the "outstanding amount of debt." So, if for example the initial loan was \$10 000 000, but the company already paid back \$9 000 000, then we will have only one million of debt.

Definition 6 (Equity)

share capital := equity = the value of the shares issued by the company = asset minus cost of liabilities

Example (Equity)

A company that has only one asset of \$100 000 and a loan against that asset (with outstanding amount of \$40 000) has \$60 000 equity.

Definition 7 (CapEx)

Capital Expenditure (CapEx) is an expense made by a company for which the benefit to the company continues over a long period (multiple accounting cycles), rather than being used and exhausted in a short period (shorter than one accounting cycle). Such expenditure is assumed to be a non-recurring nature and results in acquisition of durable assets.

Further information – Capex

CapEx is also referred to as “Capital Expense”, both are synonyms.

In accounting, one will not book CapEx as a cost but rather add them to capital and then depreciate. This allows a regular and durable reduction of taxes. For example, a rail-transport company would depreciate a train over 10 years, because it can typically be used longer.

Definition 8 (OpEx)

An Operational Expenditure (OpEx) is an ongoing and/or recurring cost to run a business/system/product/asset.

Note – Opex

OpEx is also referred to as “Operational Expense” or “Operational Cost”.

In accounting, OpEx is booked as “costs” and will reduce the taxable income of that year (except when local rules force it to be re-added for tax purposes).

For example, the diesel to run a train, its maintenance, salary costs of the driver, oil for the motor, etc. are all OpEx for the train (which would be booked as CapEx).

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SECTION 4:

Selected Financial Ratios

Definition 9 (Profit Margin (PM))

$$PM = \frac{\text{EBIT}}{\text{Sales}}$$

Gross Margin

Definition 10 (Gross Margin (GM))

$$\text{GM} = \frac{\text{Gross Profit}}{\text{Sales}}$$

Definition 11 (Asset Utilisation (AU))

$$\text{AU} = \frac{\text{Sales}}{\text{net Total Assets}} = \frac{\text{Sales}}{\text{TCstE}}$$

The liquid ratio is defined as follows.

Definition 12 (Liquid Ratio (LR))

$$
\begin{aligned}
\text{LR} &= \frac{\text{Liquid Assets}}{\text{Liquid Liabilities}} \\
&= \frac{\text{Current Assets - Stock}}{\text{Current Liabilities}} \\
&= \frac{\text{LA}}{\text{LL}}
\end{aligned}
$$

Definition 13 (Current Ratio (CR))

$$
\begin{aligned}
\text{CR} &= \frac{\text{Current Assets}}{\text{Current Liabilities}} \\
&= \frac{\text{CA}}{\text{CL}} \\
&= \text{LR}
\end{aligned}
$$

Definition 14 (Quick ratio (QR))

$$\text{QR} = \frac{\text{Cash + cash equivalents}}{\text{Liquid Liabilities}}$$
$$= \frac{\text{Current Assets - Stock}}{\text{Current Liabilities}}$$

Operating Assets

Definition 15 (Operating Assets (OA))

$$
\begin{aligned}
\text{OA} &= \text{total assets} - \text{financial assets} \\
&= \text{TA} - \text{FA} \\
&= \text{TA} - \text{cash} \qquad \text{(usually)}
\end{aligned}
$$

The OA are those assets that the company can use for its core business. For most companies (so, excluding banks and investment funds) the operating assets are all the assets minus the financial assets such as shares of other companies. [1]

For example the factory, the machines and tools would qualify as operating assets for a car manufacturer, but not the strategic stake that is held in the competing brand.

Definition 16 (Operating Assets (OL))

$$
\begin{aligned}
\text{OL} &= \text{total liabilities} - \text{financial liabilities} \\
&= \text{TL} - \text{FL} \\
&= \text{TL} - \text{short term notes} - \text{long term notes} \qquad \text{(usually)}
\end{aligned}
$$

The operating liabilities are those liabilities that the company should settle in order to pursue its regular business model.

Definition 17 (Net Operating Assets (NOA))

$$\begin{aligned} \text{NOA} &= \text{operating assets} - \text{operating liabilities} \\ &= \text{OA} - \text{OL} \end{aligned}$$

Definition 18 (Working Capital (WC))

$$\begin{aligned} \text{WC} &= \text{current assets} - \text{current liabilities} \\ &= \text{CA} - \text{CL} \end{aligned}$$

Total Capital (Employed)

Definition 19 (Total Capital Employed (TC or TCE))

$$\text{TCE} = \text{share capital} + \text{reserves} + \text{loans}$$

Note that we use TCE and TC interchangeably: $\text{TCE} := \text{TC}$.

Weighted Average Cost of Capital (WACC)

Definition 20 (Weighted Average Cost of Capital (WACC))

The cost of capital is the cost of a company's funding (debt and equity), or, from an investor's point of view "the required rate of return on a portfolio of the company's existing securities."

WACC is used to evaluate new projects of a company: it is the minimum rate of return that a new project should bring. WACC is also the minimum return that investors expect in return for capital they allocate to the company (hereby setting a benchmark that a new project has to meet – so both are equivalent).

$$\begin{aligned} \text{WACC} &= \frac{\sum_{i=1}^{N} R_i \, V_i}{\sum_{i=1}^{N} V_i} \\ &= \frac{D}{D+E} \, K_d + \frac{E}{D+E} \, K_e \qquad \text{(if only funded by equity and debt)} \end{aligned}$$

With V_i the market value of asset i, R_i the return of asset i, E the total equity, D the total debt, K_e the cost of equity, and K_d the cost of debt.

Definition 21 (Reinvestment rate (RIR))

$$\text{RIR} = \text{Reinvestment Rate} = \frac{g}{\text{ROIC}}$$

where g is the growth rate and ROIC is the return on capital.

Definition 22 (Coverage Ratio (CoverageR))

$$\text{CoverageR} = \frac{\text{Operating Income}}{\text{Financial Expenses}}$$

Definition 23 (Gearing Ratio (GR))

$$\begin{aligned} \text{GR} &= \frac{\text{Loans}}{\text{TCE}} \\ &= \frac{\text{Loans}}{\text{Shareholders Equity + Reserves + Loans}} \end{aligned}$$

Debt-to-equity ratio

Definition 24 (Debt-to-equity ratio (DE))

$$DE = \frac{\text{Loans}}{\text{Equity}}$$

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Management Accounting

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SECTION 1:

Introduction

Definition 25 (Management Accounting (MA))

Management Accounting is the provision of financial and non-financial decision-making information to managers.
According to the Institute of Management Accountants (IMA): "Management accounting is a profession that involves partnering in management decision making, devising planning and performance management systems, and providing expertise in financial reporting and control to assist management in the formulation and implementation of an organization's strategy."

Management Accounting (MA) is the section of the company that supports the management to make better informed decisions and planning support. To do so, it will use data to monitor finances, processes, and people to prepare a decision and after the decision it will help to follow up the impact.

Difference between MI and FI

Financial Accounting	Management Accounting
mainly external use (mainly for shareholders, tax, creditors)	internal use (only for management)
past oriented (shows what happened)	future oriented (decision support)
fixed reporting period (annual, quarter, month)	flexible reporting period (period as is relevant)
precise (up to £0.01)	mainly indicates direction
always in one currency	currency but other measures possible
required by law	not required by law
public information	confidential

Importance of MI

Managing a company only relying on financial accounting and without good Management Information is similar to driving a car by only using the rear mirrors. While it might work in extremely favourable conditions it is not a method that prepares one for bumpy roads ahead.

Definition 26 (MIS)

A Management Information Systems (MIS) holds operational information that that can be used to improve the efficiency and effectiveness of strategic decision making.

The concept may include transaction processing system, decision support systems, expert systems, and executive information systems. The term MIS is often used in the business schools. Some of MIS contents are overlapping with other areas such as information system, information technology, informatics, e-commerce and computer science. Therefore, the MIS term sometimes can be inter-changeable used in above areas.

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SECTION 2:

Selected Methods in MA

Definition 27 (– Cost accounting)

Cost accounting is an accounting process that measures and analyses the costs associated with products, production, and projects so that correct amounts are reported on financial statements.

Cost accounting aids in the decision-making processes by allowing a company to evaluate its costs. Some types of costs in cost accounting are direct, indirect, fixed, variable and operating costs.

- *Standard cost accounting (SCA):* Standard cost accounting (SCA) uses ratios called efficiencies that compare the labour and materials actually used to produce a good with those that the same goods would have required under "standard" conditions – works well if only labour is the main cost driver (as was the case in the 1920s when it was introduced).

- *Activity-based costing (ABC):* Activity-based costing (ABC) is a costing methodology that identifies activities in an organization and assigns the cost of each activity with resources to all products and services according to the actual consumption by each. This model assigns more indirect costs (overhead) into direct costs compared to conventional costing and also allows for the use of activity-based drivers.

- *Lean accounting:* Lean accounting is introduced to support the lean enterprise as a business strategy (the company that strives to follow the principles of Lean Production). The idea is to promote a system that measures and motivates best business practices in the lean enterprise by measuring those things that matter for the customer and the company.

- *Resource consumption accounting (RCA):* Resource consumption accounting (RCA) is a management theory describing a dynamic, fully integrated, principle-based, and comprehensive management accounting approach that provides managers with decision support information for enterprise optimization. RCA is a relatively new, flexible, comprehensive management accounting approach based largely on the German management accounting approach Grenzplankostenrechnung (GPK)

- *Grenzplankostenrechnung (GPK)*: is a German costing methodology, developed in the late 1940s and 1950s, designed to provide a consistent and accurate application of how managerial costs are calculated and assigned to a product or service. The term Grenzplankostenrechnung, often referred to as GPK, has been translated as either “marginal planned cost accounting” or “Flexible Analytic Cost Planning and Accounting”.[2]
- *Throughput accounting (TA):* Throughput accounting[3] is a principle-based and simplified management accounting approach that aims to maximize throughput[4] (sales reduced with total variable costs). It is not a cost accounting approach as it does not try to allocate all costs (only variable costs) and is only cash focused. Hence, TA tries to maximize Throughput (T): $T = S - TVC$. This throughput is typically expressed as a Throughput Accounting Ratio (TAR), defined as follows: $TAR = \frac{\text{return per factory hour}}{\text{cost per factory hour}}$.

- *Life cycle costing (LCCA):* Life-cycle cost analysis (LCCA) is a tool to determine the most cost-effective option among different competing alternatives to purchase, own, operate, maintain, and finally, dispose of an object or process, when each is equally appropriate to be implemented on technical grounds. Hence, LCCA is ideal to decide what to use and how to do it. For example, it can be used to decide which types of rails to use, which machine to use to put the rails, how to finance the machine, etc.[5]

- *Environmental accounting:* Environmental accounting incorporates both economic and environmental information. It can be conducted at the corporate level, national level or international level (through the System of Integrated Environmental and Economic Accounting, a satellite system to the National Accounts of Countries (those that produce the estimates of Gross Domestic Product (GDP))). Environmental accounting is a field that identifies resource use, measures and communicates costs of a company’s or national economic impact on the environment. Costs include costs to clean up or contain contaminated sites, environmental fines, penalties and taxes, purchase of pollution prevention technologies and waste management costs.

- *Target costing (TargetC):* Target costing is a cost management tool for reducing the overall cost of a product over its entire life-cycle with the help of production, engineering, research and design. A target cost is the maximum amount of cost that can be incurred on a product and with it the firm can still earn the required profit margin from that product at a particular selling price.[6]

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section 3:

Selected Cost Types

Definition 28 (Direct Cost)

A Direct Cost is a cost used to produce a good or service and that can be identified as directly used for the good or service.

For example: Direct Cost can be (raw) materials, labour, expenses, marketing and distribution costs if they can be traced to a product, department or project.

Definition 29 (Marginal Cost)

The Marginal Cost is the expense to produce one more unit of product. This can also be defined as $C_M = \frac{\partial P}{\partial Q}$ (with P the price, A the quantity produced and C_M the Marginal Cost.

Definition 30 (Indirect Cost)

An Indirect Cost is an expense that is not directly related to producing a good or service, and/or cannot be easily traced to a product, department, activity or project that would be directly related to the good or service considered.

Example (A Computer Assembly Facility)

An assembly facility will easily allocate all components and workers to the end-product (e.g. a specific mobile phone or tablet). However, the cost to rent the facility, the electricity and the management are not easily allocated to one type of product, so they can be treated as Indirect Costs.

Fixed Cost

Definition 31 (Fixed Cost)

A fixed cost is an expense that does not vary with the number of goods or services produced (at least in medium or short term).

Definition 32 (Variable Cost)

A Variable Cost is an expense that changes directly with the level of production output.

Example (A Computer Assembly Facility)

The lease of the facility will be a fixed cost: it will not vary in function of the number of phone and tablets produced. However, the electricity and salaries might be Variable Costs.

Definition 33 (Overhead Cost)

An Overhead Cost ("operating expense", or "overhead expense") is an on-going expense inherent to operating a business that cannot be easily traced to or identified with any particular cost unit (cost centre).

Overhead expenses can be defined as all costs on the income statement except for direct labour, direct materials, and direct expenses. Overhead expenses may include accounting fees, advertising, insurance, interest, legal fees, labour burden, rent, repairs, supplies, taxes, telephone bills, travel expenditures, and utilities. Note that Overhead Cost can be Variable Overhead (e.g. office supplies, electricity) or Fixed Overhead (lease of a building).

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SECTION 4:

Selected Use Cases of MA

Definition 34 (Balanced scorecard (BS))

The Balanced Scorecard (BSC) is a structured report that helps managers to keep track of the execution of activities, issues, and relevant measures. The critical characteristics that define a balanced scorecard are:

- its focus on the strategic agenda of the organization concerned,
- the selection of a small number of data items to monitor (that are or collections or are expected to monitor a wider concept),
- a mix of financial and non-financial data items,
- a comparison to an expected or hoped for result (closed loop controller)

Rather than just a card to measure performance, it tries to link into the strategic long-term goals; therefore, it should be composed of the following parts:

- *A destination statement.* This is a one or two page description of the organisation at a defined point in the future, typically three to five years away, assuming the current strategy has been successfully implemented. The descriptions of the successful future are segmented into perspectives for example financial & stakeholder expectations, customer & external relationships, processes & activities, organisation & culture.
- *A strategic linkage model.* This is a version of the traditional "strategy map" that typically contains 12-24 strategic objectives segmented into two perspectives, activities and outcomes, analogous to the logical framework. Linkages indicate hypothesised causal relations between strategic objectives.
- *A set of definitions for each of the strategic objectives.*
- *A set of definitions for each of the measures selected to monitor each of the strategic objectives, including targets.*

Definition 35 (KPI)

A Key Performance Indicator (KPI) is a measure used to bring about behavioural change and improve performance.

Example (Net Promoter Score (NPS) and customer satisfaction)

Our management thinks that customer engagement is key and identifies NPS[a] as a KPI (or even as *the* KPI). Doing so it engages all employees to provide customers with a better experience, better product, sharp price, after sales service, etc. Almost everything the company does will somehow contribute to this KPI.

[a] More about NPS in a few slides.

Definition 36 (Lagging Indicator)

A Lagging Indicator is an “output” indicator, it is the result of something but and by the time it is measured it is too late for management to intervene. It explains why we have today the profit at a certain level.

Example (Lagging Indicator)

All financial indicators such as profit, ROI, etc. are lagging indicators, they are the result of many actions, their impact is on the profit that we have today.

Leading KPIs

Definition 37 (Leading Indicator)

A Leading Indicator is a measure that is predicts future profit.

Example (Leading Indicator)

The textbook example would be that when your strategic goal is to live longer and be physically fitter, then most probably your level one KPI is weight loss. But that is a Lagging Indicator: when you are on the scale and you read out the weight, it is too late to do something about it. Leading indicators that feed into this lagging indicator are for example food intake, hours of workout, etc.

Note – KPIs and corporate organisation

Typically, Leading Indicators feed into Lagging Indicators. The higher up the organization chart, the more KPIs become lagging. One of the finer arts of management is to turn these lagging KPIs into actionable strategies, leading KPIs, leading actions, etc.

Customer Value Metric (CVM)

Definition 38 (Customer Value Metric (CVM))

A Customer Value Metric is an estimation of the monetary value that a customer represents.

Customer Value Metrics can be:

- *Historic:* e.g. the net profit on a customer over the last year.
- *Expected:* also referred to as Customer Lifetime Value or Lifetime Customer Value this is the present value of the total value that be expected to be derived from this customer.
- *Potential:* the maximal obtainable customer value.

Note – The usefullness of past customer profit

The answer is: “it depends”. If past income is predictable for the future income (if no cross selling or up-selling is possible), then it is important. In most cases, however, past income on a customer is not the most essential: the future income is the real goal. Banks will for example provide free accounts to youngsters and loss making loans to students in the hope that most of those customers will stay many years and become profitable.

Common Pitfalls in CVM Use and Calculation

- Use **Income or Gross Profit** in stead of Net Income.
- **Not allocating costs logically**.
- Allocating costs **politically**.
- Cost allocation is **not detailed enough**.
- Blind use of **existing customer segmentation**.
- Trust **intuition** instead of numbers.
- CVM is an output model (not an input model). If model inputs change then the CVM will change (e.g. better customer service will reduce churn).
- **Correlation** between the CVM of different segments can increase **risk**.

Subjective concepts such as customer satisfaction are best measured on some simple scale – such as "bad," "good," and "excellent." To our experience the best is to use a scale of five options: seven is too much to keep in mind, three is too simple in order to present any gradation. A "scale from one to five" is almost naturally understood by everyone.[7]

Usually, the scale is more or less as follows:

1. really bad/dissatisfied,
2. acceptable but not good/satisfied,
3. neutral,
4. good/satisfied,
5. really good/extremely satisfied.

It is now possible to define the NPS as follows:

Definition 39 (Net Promoter Score (NPS))

$$\text{NPS} := \frac{\#\text{Promoters} - \#\text{Detractors}}{\text{Total } \#\text{Customers}}$$

Where Promoters are the people that score highest possible (e.g. 5 out of 5) and Detractors are people that score lowest (1 out of 5 — though it might make sense[a] to make this range wider such as scores 1 and 2). The middle class that is not used in the nominator is called the "Passives": these are the people that do not promote us, nor

[a] In the original work one argues that "customers that give you a 6 or below (on a scale from 1 to 10) are Detractors, a score of 7 or 8 are called

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Asset Valuation Basics

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CHAPTER 30: ASSET VALUATION BASICS

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SECTION 1:

Time Value of Money

Interest

Lending an asset holds risk for the lender and is therefore, compensated by paying interest to the lender. If the asset has a value V_0 today and r is the unit interest rate over a unit period (e.g. one year), then the interest due over one period is

$$I = r\, V_0$$

So, lending an asset over one period and giving it back at the end of that period plus the interest equals paying $(1 + r)V_0$ at the end of that period. Therefore, the value of an asset within one year V_1 is:

$$V_1 = (1 + r)\, V_0$$

This value in the future is called the "future value" (FV). The FV of an asset after N years becomes then:

$$\mathrm{FV} = (1 + r)^N\, V_0$$

Question #1

If the interest rate over one year is r_y, then how much interest is due over one month?(While in general that will depend on how many days the month has, for this exercise work with $\frac{1}{12}^{\text{th}}$).

Definition 40 (APR or AER)

the Annual Percentage Rate (APR) or Effective Annual Rate or Annual Equivalent Rate (AER) is the annualized compound interest rate (r_y) that takes into account all costs for the borrower.

Consider now the following example. A loan-shark asks a 5% interest rate for a loan of one month. What is the APR? Well the APR is the annual equivalent interest rate, so we have to see what a 5% interest per week would cost us over a full year.

$$\text{APR} = r_y = (1 + r_w)^{52} - 1$$

This can be formulated in R as follows:

```
r_w <- 0.05              # the interest rate per week
r_y <- (1 + r_w)^52 - 1  # assume 52 weeks per year
paste0("the APR is: ", round(r_y * 100, 2),"%!")
## [1] "the APR is: 1164.28%!"
```

Nominal vs. Real Interest Rates

Definition 41 (Nominal Interest Rate)

The nominal interest rate (i_n) is the rate of interest (as shown or calculated) with no adjustment for inflation.

Definition 42 (Real Interest Rate)

The real interest rate (i_r) is the growth in real value (purchase power) plus interest corrected for inflation (p).

Example (Real Interest Rate)

Assume that the inflation p is 10% and you borrow $100 for one year and the lender asks you to pay back $110 after one year. In that case, you pay back the same amount in real terms as the amount that you have borrowed, so the real interest rate is 0%, while the nominal interest rate is 10%.

The Net Present Value (NPV) is the Future value discounted to today:

$$\text{PV} = \frac{\text{FV}}{(1+r)^N} \tag{1}$$

Hence, the Net Present Value of a series of cash flows (CF) equals:

$$\text{NPV} = \sum_{t=0}^{N} \frac{\text{CF}_t}{(1+r)^t} \tag{2}$$

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section 2:

Cash

Definition 43 (Cash)

The strict definition of Cash is money in the physical form of a currency, such as banknotes and coins.
In bookkeeping and finance, cash refers to current assets comprising currency or currency equivalents that can be converted to cash (almost) immediately. Cash is seen either as a reserve for payments, in case of a structural or incidental negative cash flow, or as a way to avoid a downturn on financial markets.

Example (Cash)

For example, typically one considers current accounts, savings accounts, short term Treasury notes, etc. also as "cash."

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SECTION 3:

Bonds

Definition 44 (Bond)

In finance, a bond is an instrument of indebtedness of the bond issuer to the holders. It is a debt security, under which the issuer owes the holders a debt and, depending on the terms of the bond, is obliged to pay them interest (the coupon) and/or to repay the principal at a later date, termed the maturity date. Interest is usually payable at fixed intervals (semi-annual, annual, sometimes monthly). A bond is also transferable. That means that the obligor owns money to the holder of he bond and not to the initial holder.

Definition 45 (Principal)

Nominal, principal, par, or face amount is the amount on which the issuer pays interest, and which – usually – has to be repaid at the end of the term. Some structured bonds can have a redemption amount which is different from the face amount and can be linked to performance of particular assets.

Definition 46 (Maturity)

The issuer has to repay the nominal amount on the maturity date. As long as all due payments have been made, the issuer has no further obligations to the bond holders after the maturity date. The length of time until the maturity date is often referred to as the term or tenor or maturity of a bond. The maturity can be any length of time. Most bonds have a term of up to 30 years, however, some issues have no maturity date (“irredeemables” or “eternal bonds”).

In the market for United States Treasury securities, there are usually three categories of bond maturities considered:

- *Short term (bills):* Maturities between one to five year; (instruments with maturities less than one year are called Money Market Instruments).

- *Medium term (notes):* Maturities between six to twelve years.
- *Long term (bonds):* Maturities greater than twelve years.

Definition 47 (Coupon)

The coupon is the interest rate that the issuer pays to the holder. Usually, this rate is fixed throughout the life of the bond. It can also vary with a money market index, such as LIBOR.

Definition 48 (Yield)

The yield is the rate of return received from investing in the bond. It usually refers either to

- the current yield, or running yield, which is simply the annual interest payment divided by the current market price of the bond (often the clean price), or to
- the yield to maturity or redemption yield, which is a more useful measure of the return of the bond, taking into account the current market price, and the amount and timing of all remaining coupon payments and of the repayment due on maturity. It is equivalent to the internal rate of return of a bond.

Definition 49 (Credit quality)

The quality of the issue refers to the probability that the bondholders will receive the amounts promised at the due dates. This will depend on a wide range of factors. High-yield bonds are bonds that are rated below investment grade by the credit rating agencies. As these bonds are more risky than investment grade bonds, investors expect to earn a higher yield. These bonds are also called junk bonds.

Definition 50 (Market price)

The market price of a trade-able bond will be influenced amongst other things by the amounts, currency and timing of the interest payments and capital repayment due, the quality of the bond, and the available redemption yield of other comparable bonds which can be traded in the markets.

$$P_{\text{bond}} = \sum_{i=0}^{N} \frac{\text{CF}_i}{(1+r_i)^{t_i}} \quad (3)$$

$$= \sum_{t=1}^{N} \frac{\text{coupon}_i}{(1+r_i)^{t}} + \frac{\text{nominal}}{(1+r_N)^{N}} \quad (4)$$

for a bond that pays an annual coupon.

Consider the following example. A bond with nominal value of $100 pays annually a coupon of $5 (with the first payment in exactly one year from now), during 4 years and in the fifth year the debtor will pay the last $5 and the nominal value of $100. What is its fair price given that the risk free interest rate is 3%?
[0.5ex]1pt2mm
To solve this question, we will first create a function to calculate the price of a bond:

```
# bond_value
# Calculates the fair value of a bond
# Arguments:
#    time_to_mat -- time to maturity in years
#    coupon      -- annual coupon in $
#    disc_rate   -- discount rate (risk free + risk premium)
#    nominal     -- face value of the bond in $
# Returns:
#    the value of the bond in $
bond_value <- function(time_to_mat, coupon, disc_rate, nominal){
  value  <- 0
  # 1/ all coupons
  for (t in 1:time_to_mat) {
     value <- value + coupon * (1 + disc_rate)^(-t)
     }
  # 2/ end payment of face value
  value <- value + nominal * (1 + disc_rate)^(-time_to_mat)
  value
}
```

Now we are ready to calculate the price of the bond if we assume that the required interest rate is the same as the risk free interest rate: 3%.

```
# The fair value of the bond is then:
bond_value(time_to_mat = 5, coupon = 5, disc_rate = 0.03,
           nominal = 100)
## [1] 109.1594
```

The Macaulay duration – named after Frederick Macaulay – is the weighted average maturity of the cash flows.

$$\begin{aligned}
\text{MacD} &:= \frac{\sum_{i=0}^{N} t_i \text{PV}_i}{\sum_{i=0}^{N} \text{PV}_i} \\
&= \frac{\sum_{i=0}^{N} t_i \text{PV}_i}{V_{\text{bond}}} \\
&= \frac{1}{V_{\text{bond}}} \sum_{i=0}^{N} \frac{t_i \text{CF}_i}{(1+r_i)^{t_i}}
\end{aligned}$$

Example (Macaulay Duration)

Using the same example as aforementioned: a bond with nominal value of $100, an annual coupon of $5 (with the first payment in exactly one year from now), with a maturity of five years so that in the fifth year the debtor will pay the last $5 and the nominal value of $100. What is its fair price given that the risk free interest rate is 3%? [0.5ex]1pt2mm

```
V <- bond_value(time_to_mat = 5, coupon = 5, disc_rate = 0.03,
                nominal = 100)
CFs <- c(seq(5, 5, length.out=4), 105)
t   <- c(1:5)
r   <- 0.03
MacD <- 1/V * sum(t * CFs / (1 + r)^t)
print(MacD)
## [1] 4.56806
```

Modified Duration

$$\text{ModD} := \frac{1}{V}\frac{\partial V}{\partial y} = -\frac{\partial \log(V)}{\partial y}$$

PART 06: INTRODUCTION TO COMPANIES

↓

CHAPTER 30: ASSET VALUATION BASICS

↓

SECTION 4:

The Capital Asset Pricing Model (CAPM)

The CAPM was introduced by **??**, **?**, and **?**.[8] It is therefore, also called the "Sharpe-Lintner-Mossin mean-variance equilibrium model of exchange" – the Capital Asset Pricing Model (CAPM) – is used to determine a theoretically appropriate required rate of return of an asset in function of that asset's non-diversifiable risk and the inherent risk to the market. The model takes into account the asset's sensitivity to non-diversifiable risk (also known as systemic risk or market risk), often represented by the quantity beta (β) in the financial industry, as well as the expected return of the market, and the expected return of a theoretical risk free asset.

CAPM — the Model (1)

The CAPM is expressed as follows:

$$\frac{E[R_k] - R_{\mathrm{RF}}}{\beta_k} = E[R_M] - R_{\mathrm{RF}} \tag{5}$$

The market reward-to-risk ratio is effectively the market risk premium. Rearranging the aforementioned equation and solving for $E(R_k)$, we obtain the expected return from the asset k via the CAPM:

$$E[R_k] = R_{\mathrm{RF}} + \beta_k \left(E[R_M] - R_{\mathrm{RF}}\right) \tag{6}$$

where:

- $E[R_k]$ is the expected return on the capital asset.
- R_{RF} is the risk free rate of interest, such as interest arising from government bonds.
- β_k (the beta coefficient) is the sensitivity of the asset returns to market returns, or also $\beta_k = \frac{\mathrm{Cov}(R_k, R_M)}{\mathrm{VAR}(R_M)}$.
- $E[R_M]$ is the expected return of the market.
- $E[R_M] - R_{\mathrm{RF}}$ the market premium or risk premium.
- $\mathrm{VAR}(R_M)$ is the variance of the market return.

Restated in terms of risk premium:

$$E[R_k] - R_{\text{RF}} = \beta_k \left(E[R_M] - R_{\text{RF}}\right) \tag{7}$$

which states that the individual risk premium equals the market premium times beta.

Example (Company A)

The company “A Plc.” has a β of 1.25, the market return is 10% and the risk free return is 2%.
What is the expected return for that company?
[0.5ex]1pt2mm
Since beta and R_{RF} are given, we can use the CAPM to calculate the required rate of return R_A as follows:

$$E[R_A] = R_{\text{RF}} + \beta_A \ (E[R_M] - R_{\text{RF}})$$

```
R_A <- 0.02 + 1.25 * (0.10 - 0.02)
print(paste0('The RR for company A is: ',
          round(R_A, 2) * 100, '%'))
## [1] "The RR for company A is: 12%"
```

Example (Company B)

The company "B" has a β of 0.75 and all other parameters are the same (the market return is 10% and the risk free return is 2%).
What is the expected return for that company?
[0.5ex]1pt2mm

```
R_B <- 0.02 + 0.75 * (0.10 - 0.02)
print(paste0('The RR for B is: ', round(R_B, 2) * 100, '%'))
## [1] "The RR for B is: 8%"

# Additionally, we also compare this with previous example:
print(paste0('The beta changed by ',
             round((0.75 / 1.25 - 1) * 100, 2),
             '% and the RR by ',
             round((R_B / R_A - 1) * 100, 2), '%.'))
## [1] "The beta changed by -40% and the RR by -33.33%."
```

CAPM — Risk and Diversification

Thinking of a portfolio of assets that are quoted on a stock exchange (or available on a given market), it makes sense to break down the risk of the portfolio in the two following components.

1. *Systematic risk or undiversifiable risk:* cannot be diversified away – it is inherent to the market under consideration (“market risk”).
2. *Unsystematic risk, idiosyncratic risk or diversifiable risk:* the risk of individual assets. Unsystematic risk can be reduced by diversifying the portfolio (specific risks “average out”).

CAPM - about Risk

- *A rational investor should not take on any diversifiable risks:* Therefore the required return on an asset (i.e. the return that compensates for risk taken), must be linked to its riskiness in a portfolio context– i.e. its contribution to the portfolio's overall riskiness – as opposed to its "stand-alone riskiness."
- In CAPM, *portfolio risk is represented by variance*. Therefore the beta of the portfolio is the defining factor in rewarding the systematic exposure taken by an investor.
- The CAPM assumes that the volatility-return profile of a *portfolio can be optimized as in Mean Variance Theory*.
- Because the unsystematic risk is diversifiable, the *total risk of a portfolio can be viewed as beta*.

CAPM — Assumes that all investors . . .

1. try to maximize utility that is a function of only return and volatility,
2. have a stable utility function (does not depend on the level of wealth),
3. are rational and volatility-averse,
4. consider all assets in one portfolio,
5. do not care about other live goals apart from money (investments are a life goal in their own right and do not serve to cover other liabilities or goals),
6. are price takers, i.e. they cannot influence prices,
7. are able to lend and borrow under the risk free rate of interest with no limitations,
8. trade without transaction costs,
9. are not taxed in any way on their investments or transactions,
10. deal with securities that are all highly divisible into small units, and
11. assume all information is at the same time available to all investors.

PART 06: INTRODUCTION TO COMPANIES

↓

CHAPTER 30: ASSET VALUATION BASICS

↓

SECTION 5:

Equities

Definition 51 (Stock, shares and equity)

A share in a company is a title of ownership. The capital stock of an incorporated business constitutes the equity stake of its owners. It represents the residual assets of the company that would be due to stockholders after discharge of all senior claims such as secured and unsecured debt.

There are some different classes of shares that are quite different.

1. **Common stock** usually entitles the owner to vote at shareholders' meetings and to receive dividends.
2. **Preferred stock** generally does not have voting rights, but has a higher claim on assets and earnings than the common shares. For example, owners of preferred stock receive dividends before common shareholders and have priority in the event that a company goes bankrupt and is liquidated.

Digression – Local use of definitions

In some jurisdictions such as the United Kingdom, Republic of Ireland, South Africa, and Australia, stock can also refer to other financial instruments such as government bonds.

- *Roman Republic*, the state outsourced many of its services to private companies. These government contractors were called *publicani*, or *societas publicanorum* (as individual company). These companies issued shares called *partes* (for large cooperatives) and *particulae* for the smaller ones.[9]
- *ca. 1250:* 96 shares of the *Société des Moulins du Bazacle* were traded (with varying price) in Toulouse
- *31/12/1600:* the East India Company was granted the Royal Charter by Elizabeth I (earliest recognized joint-stock company in modern times)[10]
- *1602:* the "Vereenigde Oostindische Compagnie" issued shares that were traded on the Amsterdam Stock Exchange
- The invention of the stock exchange made pooling of capital efficient and allowed for larger financial expenses such as building ships . . . the success of The Netherlands as a maritime superpower soon followed
- Dutch stock market of the *17th century* had
  - stock futures,
  - stock options,
  - short selling,
  - credit to purchase stock (margin trading or "trading on a margin"),
  - . . . and the Tulipomania in 1637 – **?**

1. *Absolute value models* try to predict future cash flows and then discount them back to the present value.
2. *Relative value models* rely on the collective wisdom of the financial markets and determine the value based on the observation of market prices of similar assets.

There is a lot to valuation of equities, in this first approach we assume simply a flow of dividends and ask us what this flow of dividends should be worth.

$$P_{\text{equity}} = \sum_{t=0}^{N} \frac{\text{CF}_t}{(1+r)^t} \tag{8}$$

$$= \sum_{t=0}^{\infty} \frac{D_t}{(1+r)^t} \tag{9}$$

with $D = \textit{dividend}$.

The Dividend Discount Model (DDM)

Theorem 1 (DDM).

The value of a stock is given by the discounted stream of dividends:

$$V_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+r)^t}$$

Capital gains appear as expected sales value and are derived from expected dividend income.

- V_0 the intrinsic value of the stock now
- D_t the dividend paid in year t
- r is the capitalization rate and is the same as $E[R_k]$ in the CAPM, see Equation 6 on slide 87

If every year the dividend increases with the same percentage, $100g\%$ (with g the growth rate), then each dividend can be written in function of the previous one.

$$
\begin{aligned}
D_1 &= D_0(1+g) \\
D_2 &= D_1(1+g) &&= D_0(1+g)^2 \\
&\ldots \\
D_n &= D_{n-1}(1+g) &&= D_0(1+g)^n
\end{aligned}
$$

Theorem 2 (constant-growth DDM).

Assume that $\forall t : D_t = D_0(1+g)^t$*, then the DDM collapses to*

$$V_0 = \frac{D_0(1+g)}{r-g} = \frac{D_1}{r-g}$$

Example (ABCD with $g = 0\%$)

Consider the company, ABCD. It pays now a dividend of €10 and we believe that the dividend will grow at 0% per year. The risk free rate (on any horizon) is 1%, and the market risk premium is 5% and the β is 1. What is the intrinsic value of the company?
[0.5ex]1pt2mm
Using the CGDDM, $V_0 = \frac{D_0(1+g)}{R_{ABCD}-g}$, and the CAPM $R_{ABCD} = R_{\mathrm{RF}} + \beta.RP_M$ we get:

```
V_0 <- 10 * (1 + 0.00) / (0.01 + 0.05 - 0.00)
print(round(V_0,2))
## [1] 166.67
```

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```
V_0 <- 10 * (1 + 0.00) / (0.01 + 0.05 - 0.00)
print(round(V_0,2))
## [1] 166.67
```

The company value that assumes a zero growth rate is called the “no-growth-value.” Unless when the outlook is really bleak, this is seldom a valid assumption. The concept of investing in a company and hence accepting its increased risk, depends on the fact that it should – on average – be a better investment than investing in bonds or safer interest bearing products.

Example (ABCD with $g = 2\%$)

Expected growth rate of the dividend is 2%, ceteris paribus.
[0.5ex]1pt2mm

```
V_0 <- 10 * (1 + 0.02) / (0.01 + 0.05 -0.02)
print(round(V_0,2))
## [1] 255
```

Example (ABCD with $g = 2\%$)

Expected growth rate of the dividend is 2%, ceteris paribus.
[0.5ex]1pt2mm

```
V_0 <- 10 * (1 + 0.02) / (0.01 + 0.05 -0.02)
print(round(V_0,2))
## [1] 255
```

Example (ABCD with $g = 2\%$)

Expected growth rate of the dividend is 2%, ceteris paribus.
[0.5ex]1pt2mm

```
V_0 <- 10 * (1 + 0.02) / (0.01 + 0.05 -0.02)
print(round(V_0,2))
## [1] 255
```

The difference in value compared to the previous example is called the PVGO (present value of growth opportunities). So,

$$P_{\text{equity}} = \text{no-growth-value} + \text{PVGO} \quad (10)$$

$$V_0 = \frac{D_0}{r} + \text{PVGO} \quad (11)$$

Example (ABCD with $g = 2\%$ and $\beta = 1.5$)

The β is assumed to be 1.5, ceteris paribus. What is the value of ABCD?
[0.5ex]1pt2mm

```
V_0 <- 10 * (1 + 0.02) / (0.01 + 1.5 * 0.05 - 0.02)
print(round(V_0,2))
## [1] 156.92
```

Since the company is more risky and all other things remained the same, the price must be lower. For the price to be the same, investors would expect higher returns.

Example (ABCD — extreme growth)

The dividend growth rate is now expected to be 10%, ceteris paribus.
[0.5ex]1pt2mm
Simply adding that growth rate in the formula leads to impossible results:

```
V_0 <- 10 * (1 + 0.02) / (0.01 + 1.5 * 0.05 - 0.10)
print(round(V_0,2))
## [1] -680
```

A companies equity can never become negative, because the equity holder is only liable up to the invested amount. Therefore this result is not possible: it indicates a situation where the DDM fails (the growth rate cannot be larger than the required rate of return).

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[0.5ex]1pt2mm
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print(round(V_0,2))
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```

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Example (ABCD — extreme growth)

The dividend growth rate is now expected to be 10%, ceteris paribus.
[0.5ex]1pt2mm
Simply adding that growth rate in the formula leads to impossible results:

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print(round(V_0,2))
## [1] -680
```

A companies equity can never become negative, because the equity holder is only liable up to the invested amount. Therefore this result is not possible: it indicates a situation where the DDM fails (the growth rate cannot be larger than the required rate of return).

This example illustrates that the DDM is only valid for dividend growth rates smaller than the required rate of return. A company that would grow faster, would be deemed to be more risky and hence would have a higher beta, leading to a higher discount rate. The model states that anything above that is unsustainable and will lead to a correction.

Relationship between growth rate and ROE

To better understand how the growth rate related to certain accounting values, we introduce the following concepts.

Definition 52 (earnings)

$E := \text{net income}$

Definition 53 (dividend payout ratio)

$\text{DPR} := \frac{D}{E}$

Definition 54 (plow-back ratio (earnings retention ratio))

$\text{PBR} := 1 - \text{DPR}$

Definition 55 (Return on Equity)

$\text{ROE} := \frac{E}{P}$

the growth rate of the dividends is related to the DPR.

$$g = ROE \times PBR \tag{12}$$

This is because if the company retains x% earnings, then the next dividend will be x% higher. More generally:

$$g = \frac{\text{reinvested earnings}}{\text{BV}} = \frac{\textit{reinvested earnings}}{\text{TE}} \frac{\text{TE}}{\text{BV}}$$

Hence, – as mentioned earlier–

$$P_{\text{equity}} = \text{no-growth-value} + \text{PVGO} \tag{13}$$

$$P_0 = \frac{D_0}{r} + \text{PVGO} \tag{14}$$

the growth rate of the dividends is related to the DPR.

$$g = ROE \times PBR \tag{12}$$

This is because if the company retains x% earnings, then the next dividend will be x% higher. More generally:

$$g = \frac{\text{reinvested earnings}}{\text{BV}} = \frac{\textit{reinvested earnings}}{\text{TE}} \frac{\text{TE}}{\text{BV}}$$

Hence, – as mentioned earlier–

$$P_{\text{equity}} = \text{no-growth-value} + \text{PVGO} \tag{13}$$

$$P_0 = \frac{D_0}{r} + \text{PVGO} \tag{14}$$

If the stock trades at its intrinsic value (i.e. $P_0 = V_0$) and if we assume the CGDDM then

$$P_0 = \frac{D_1}{r - g} \tag{15}$$

Advantages

- logical and complete in a liquid market,
- easy to understand, and
- it only makes assumptions about the outcome (dividend) and not the thousands of variables that influence this variable.

Disadvantages

- to forecast an infinite amount of dividends,
- to find a good discount rate (which is complex and actually circular), and hence
- it is incomplete without stress testing the result.

The first thing to do is to identify the cash that is available to pay dividends. It is reasonable to assume that the only value a company brings to the shareholder is the financial income. This means that the value of the company is the present value of all future cash flows to the equity holder paid up-front (now and at once). Therefore, it is important to use only cash flows that go to the shareholder and exclude salaries, bonuses, taxes, etc. . . . so we use free cash flow (FCF).

Definition 56 (Free Cash Flow (FCF))

Free Cash Flow is the cash flow available for distribution to equity holders of the company.

$$\text{FCF} = \text{EBIT}(1 - \tau) + \text{Depreciation} + \text{Amortization} - \Delta\text{WC} - \text{CapEx}$$

Alternative Ways to Calculate FCF

$$
\begin{aligned}
\text{FCF} = &\quad \text{EBIT}\,(1-\tau) + \text{Depreciations} + \text{Amortisations} - \Delta\text{WC} - \text{CapEx} \\
= &\quad \text{NOPAT} + \text{Interest Expense} + \text{Depreciations} + \text{Amortisations} \\
&\quad -\Delta\text{WC} - \text{CapEx} - \text{Tax Shield On Interest Expense} \\
= &\quad \text{PAT} - (1-d)\,(\text{Depreciations} + \text{Amortisations} - \Delta\text{WC} - \text{CapEx})
\end{aligned}
$$

Definition 57 (Discounted Cash Flow)

Discounted cash flow (DCF) is a method of valuing a company, project, or any other asset by discounting future cash flow to today's value (in other words: using the time value of money) and then summing them.

Definition 58 (NPV)

The Net Present Value (NPV) is then the sum of all present values and represents today's value of the asset.

The DCF model in company valuation is simply calculating the NPV of the companies FCF.

DCF Model for Company Valuation

For company valuation we will substitute:

- *CF* by FCF, because that is the relevant cash flow for the potential buyer of the company.
- *r* by WACC, because the company should at least make good for compensating its capital. needs

Just as for the DDM it is possible to make some simplifying assumptions:

$$
\begin{aligned}
V &= \sum_{t=1}^{\infty} \frac{\mathrm{FCF}_t}{(1+\mathrm{WACC})^t} \\
&= \sum_{t=1}^{\infty} \frac{\mathrm{FCF}_{t-1}(1+g_t)}{(1+r)^t} && \text{(with } FCF_0 \text{ known)} \\
&= \mathrm{FCF}_0 \sum_{t=1}^{\infty} \frac{(1+g)^t}{(1+\mathrm{WACC})^t} && \text{(assuming } \forall g_t : g_t = g\text{)} \\
&= \mathrm{FCF}_0 \frac{1+g}{\mathrm{WACC}-g} && \text{(assuming } g < \mathrm{WACC} \text{ and } g \neq -1\text{)} \\
&= \frac{\mathrm{FCF}_0}{\mathrm{WACC}} && \text{(assuming } g = 0\text{)}
\end{aligned}
$$

Advantages of the DCF method

- Always applicable to all companies.
- Logical and complete.
- Easy to understand.
- DCF allows for the most detailed view on the company's business model.
- It can be used to model synergies and/or influence on the company's strategy.

Disadvantages of the DCF method

- Determining the FCF is not too easy.
- One needs to forecast an infinite amount of FCFs (and therefore, one needs to model the whole balance sheet).
- Therefore, one needs many assumptions (costs, inflation, labour costs, sales, tax rates, etc.)
- One needs to find a good discount rate (which is complex and actually circular).
- It is incomplete without stress testing the result.

Net Asset Value Method (NAV Method)

Definition 59 (Net Asset Value Method)

The Net Asset Value Method is also known as the Liquidation Method. The idea is to use the liquidation value as a proxy for the company's value. The question one answers is: "if the company stopped trading now, what would be got from all assets (reduced with all liquidation costs)."

Question #2

Investment Funds (aka Mutual Funds) are always valued with the NAV method. Why?

Question #3

Investment Funds (aka Mutual Funds) are always valued with the NAV method. Why?

Answer

Investment funds have no other operations than investing in the stock exchange (or other assets such as bonds, CDOs, notes, etc. Since there are no other profitable operations there is nothing else than the assets.

Question #4

The typical investment fund will value assets at market value (as opposed to book value). Why?

Question #5

The typical investment fund will value assets at market value (as opposed to book value). Why?

Answer

If and investor would redeem his/her holding from the fund and the fund were to sell the investment held in portfolio, then it would get "market value." This market value is in its turn the total net present value of all cash flows that can be expected from these assets. So, – if the markets are liquid enough – then the market value is the relevant value.

Advantages

- Simple and straightforward.
- Easy to understand.
- No assumption needed about a discount rate.
- It is all we need for companies in receivership and investment funds.

Disadvantages

For normal operating companies that are not in receivership:

- The NAV is only the lower limit of the real value (and hence merely a reality check) and for normal companies it misses the point of a valuation.
- It is irrelevant for growth companies (e.g. Google, Uber, Facebook).

Definition 60 (Price or market value)

$P_0 =$ the price paid for a company on the market (market price).

Definition 61 (Value)

$V_0 =$ the real value of a company (intrinsic value).

A short-cut: the price is the consensus of the market about the value.

Definition 62 (Market capitalization)

The market capitalisation (often shortened to “market cap”) is the total value of all outstanding stocks at the market price. It is the value of the company as fixed by the market.

Price Earnings Ratio (PE)

Definition 63 (– Price earnings ratio (PE ratio))

The price earnings ratio is the market price of a company divided by its last annual earnings.

$$\text{PE} := \frac{V_0}{E_0}$$

Using this definition and rearranging Equation 14 on slide 112 shows that:

$$\frac{V_0}{E_1} = \frac{1}{r} + \frac{\text{PVGO}}{E} \tag{16}$$

$$= \frac{\text{DPR}}{r - \text{ROE} \times \text{PBR}} \tag{17}$$

$$= \frac{\text{DPR}}{r - g} \tag{18}$$

$$= \frac{1 - \text{PBR}}{r - g} \tag{19}$$

where we remind that DPR is the dividend payout ratio, PBR the plow-back ratio, PVGO is the present value of growth opportunities, r is the required discount rate and g is the growth.

Pitfalls of PE-Analysis

- E $\leftarrow$ accounting $\leftarrow$ funny rules
- Earnings management $\leftarrow$ too much freedom in accounting rules
- the relevance is limited to the model $\leftarrow$ smooth evolution of earnings (economic earnings are not measurable, the accounting earnings are)
- What about the economic cycles?
- In the formula is E_1, in reality one uses E_{-1}
- PE ratios include the future growth potential and the riskiness in one measure $\rightarrow$ one must compare likes with likes $\rightarrow$ has clear relevance within one sector
- PE ratios include the future growth potential and the riskiness in one measure $\rightarrow$ so they will jump up when the economic cycle is on its low in short term.

The price-to-book ratio is similar, but uses the book value instead of the market value of the share.

Definition 64 (Price-to-book ratio (PTB))

$\mathrm{PTB} := \frac{P}{\mathrm{BV}}$ with $\mathrm{BV} =$ book value

The book value (BV), is the value of the company as per accounting standards – in other words the size of the balance sheet. The advantage of using the book value is of course its stability, the downside is that accounting rules are designed to collect taxes. That makes accounting rules inherently backwards looking, while company valuation is essentially forward looking.

The list of possible ratios to use is only limited by your imagination and can differ for different companies. For example a steel producer can build up a stock of completed products, in a bank that works differently: they need assets to compensate risk.

Definition 65 (Price-to-cash-flow ratio (PTCF))

$\mathrm{PTCF} := \frac{P}{\mathrm{CF}}$ with $\mathrm{CF} =$ (free) cash flow

Sales is important for any company and easy to isolate in the balance sheet.

Definition 66 (Price-to-sales ratio (PTS))

$\text{PTS} := \frac{P}{S}$ with $S = \text{sales}$

Definition 67 (Return on Invested Capital (ROIC))

$$
\begin{aligned}
\text{ROIC} &= \frac{\text{Operating Profit}(1 - \text{tax rate})}{\text{Book value of Invested Capital}_{t-1}} \\
&= \frac{\text{Operating Profit} - \text{Adjusted Taxes}}{\text{Invested Capital}} \\
&= \frac{\text{EBIT}}{\text{Fixed Assets} + \text{Intangible Assets} + \text{CA} - \text{CL} - \text{cash}} \\
&= \frac{\text{EBIT}}{\text{debt + equity - cash (- goodwill)}} \\
&= \frac{\text{EBIT}}{\text{TCE}}
\end{aligned}
$$

Definition 68 (Return on Captial Employed (ROCE))

$$\text{ROCE} = \frac{\text{net operating profit}}{\text{total capital employed}} = \frac{\text{NOP}}{\text{TCE}}$$

where NOP is the net operating profit.

Definition 69 (Return on Equity (ROE))

$$\begin{aligned} \text{ROE} &= \frac{\text{Net Income}_t}{\text{equity}} \\ &= \frac{\text{Net Income}}{S} \times \frac{S}{\text{TA}} \times \frac{\text{TA}}{\text{equity}} \qquad \text{(DuPont Formula)} \\ &= \frac{\text{NOPAT}}{\text{equity}} \end{aligned}$$

where S is the sales and TA are the total assets

Note – Difference ROIC and ROE

The main difference with ROIC is that ROE does not include debt (including loans, bonds and overdue taxes) in the de-numerator. The second difference is that ROE uses in the numerator earnings after taxes (but before dividends), where RoC uses earnings before interest and taxes (EBIT).

Definition 70 (Economic Value Added (EVA))

Economic Value Added (EVA) is an estimate of the company's economic profit (the value created in excess of the required return of the company's shareholders). In other words, EVA is the net profit less the opportunity cost for the firm's capital.

$$\begin{aligned} \text{EVA} &= (\text{ROIC} - \text{WACC})\,(\text{TA} - \text{CL}) \qquad (20) \\ &= \text{NOPAT} - \text{WACC}\,(\text{TA} - \text{CL}) \qquad (21) \end{aligned}$$

Definition 71 (Market Value Added (MVA))

Market value added (MVA) is the difference between the company's current market value and the capital contributed by investors.

$$MVA = V_{\text{market}} - K \tag{22}$$

with V_{market} the market value and K the capital paid by investors.

If a company has a positive MVA this means that it has created value (in case of a negative MVA it has destroyed value). However, to determine if the company has been a good investment one has to compare the return on the invested capital with the return of the market (r_M), adjusted for the relative risk of that company (its β).

Valuation Method Selection

1. Is it my purpose to buy the company and stop its activities or did it already stop trading or is it an investment fund? – If yes, use NAV method (in all other cases this should be the lower limit). If not, then continue to next question.
2. Will you be an important share holder and can you make a business plan? – If yes, try to use DCF, if not continue.
3. Do you have the option not to invest? – If yes, use DDM otherwise continue.
4. If you ended up here, this means that you have to invest anyhow in similar stocks (e.g. you are an equity fund manager and need to follow your benchmark). – In this case, you might want to use a relative value method.

- *relevance of history:* is the past data relevant for the future?
- *short history:* it might be easier to forecast mature companies with long, stable history. Buying a company with only a few years history is a leap of confidence.
- *management differences:* will you attribute cash differently? Is the salary that the owner (not) took relevant for your case? etc.

part 06: Introduction to Companies

↓

chapter 30: Asset Valuation Basics

↓

section 6:

Forwards and Futures

Definition 72 (Future)

A future is an agreements to sell or buy at a future date at market price at that moment, when the agreement is quoted on a regulated stock exchange.

Definition 73 (Forward)

A forward an agreements to sell or buy at a future date at market price at that moment, when the agreement is an OTC agreement.

The value of a forward or future at maturity is the difference between the delivery price (K) and the spot price of the underlying as at maturity (S_T)

- For a long position, the payoff is: $F_T = S_T - K$.
- For a short position, the payoff is: $F_T = K - S_T$.

$$F_0 = S_0 \, e^{rT}$$

with T the time to maturity and r the continuously compounded risk free interest rate.

This formula can be modified to include income. If an asset pays income then this advantage is for the holder of the assets and hence we subtract it for the cash portfolio where one does not have the asset. For example, if the income is known to be a discrete series of income I_t then the formula becomes

$$F_0 = \left(S_0 - \sum_{t=1}^{N} \mathrm{PV}(I_t) \right) \exp(r\, T)$$

A continuous stream of income ι can be modelled as follows:

$$F_0 = S_0 \exp\left[(r - \iota)T\right]$$

One particular forward is the Forward Rate Agreement (FRA) that fixes an interest rate for a future transaction such as borrowing or lending. They are used to hedge against interest rate changes.

part 06: Introduction to Companies

↓

chapter 30: Asset Valuation Basics

↓

section 7:

Options

Definition 74 (Call Option)

A Call Option is the right to buy the underlying asset at a given price (the Strike) at some point in the future (the maturity date).

Definition 75 (Put Option)

A Put Option is the right to sell the underlying asset at a given price (the Strike) at some point in the future (the maturity date).

Definition 76 (Strike or Execution Price)

the “strike” or “execution price” is the price at which an option can be executed (e.g. for a call the price at which the underlying can be sold when executing the option)
The strike price is denoted as X.

Definition 77 (Maturity)

The “maturity date” is the expiry date of an option, that is the last moment in time that it can change value because of the movement of the underlying.

Definition 78 (Exercising an Option)

The act of buying or selling the underlying asset via the option contract.

Definition 79 (Delivering of the Underlying)

Providing or accepting the underlying from the option buyer who exercises his/her option.

Definition 80 (Cash Settlement)

Simply pay out the profit of the option to the buyer in cash in stead of delivering the asset.

An option is "written" by the seller and bought by the buyer.

Definition 81 (short position)

The option writer has the **obligation** to sell or buy at the pre-agreed price. He/she has a **short** position.

More Definitions ii

Definition 82 (long position)

The option buyer has the **right** to sell or buy at the pre-agreed price. He/she has a **long** position.

Definition 83 (European Option)

A European Option is an option that can be executed by the buyer at the maturity date and only at the maturity date.

Definition 84 (American Option)

An American Option is an option that can be executed by the buyer from the moment it is bought and till the at maturity date.

Imagine two call options on KBC Group NV

- option A has a strike of EUR 40, and
- option B has a strike of EUR 60

The actual price of the underlying, KBC Group NV, is EUR 50.
Which option is worth most?

Definition 85 (Spot Price)

The actual value of the underlying asset (in the sense of "today's value"), the price to be paid for the asset to buy it today and have it today.
The spot price is traditionally denoted as S.

Definition 86 (Intrinsic Value)

The value that he option will have at maturity (not discounted, just nominal value). For example,

- $IV_{call} = max(S - X, 0)$
- $IV_{put} = max(X - S, 0)$

Definition 87 (ITM)

An option is in-the-money if its Intrinsic Value is positive. So if the price of the underlying would be the same at maturity date, the option buyer would get some payoff.

Definition 88 (ATM)

An option is at-the-money if its Intrinsic Value is zero. For a call, this means that $S = X$

Definition 89 (OTM)

An option is out-of-the-money if the spot price is not equal to the strike and the intrinsic value of the option is zero. For a call, this means that $S < X$. This would mean that if at maturity the spot price would be the same as now, then the buyer would get no payoff.

Definition 90 (MTM)

A financial instrument is said to be Marked-to-Market is it is valued at its market price.

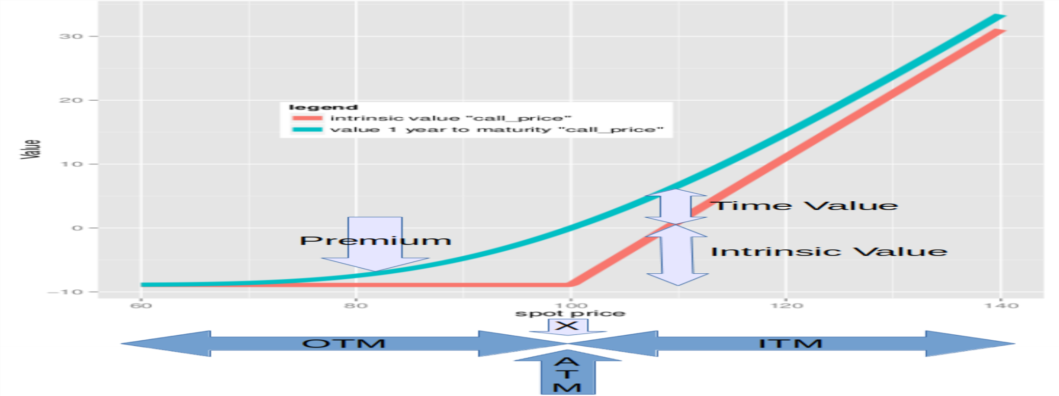


Figure 3: Some concepts illustrated on the example of a call option with on the z-axis the sport price S and on the y-axis the payoff of the structure. The length of the blue arrows illustrates the concept, while the grey arrows indicate the position of the concept on the z-axis.

An option can be bought or sold on a regulated stock exchange or “over the counter”

Definition 91 (OTC)

A financial instrument is said to be bought/sold Over the counter if it is bought/sold **outside** a regulated stock exchange.

The main differences between OTC and Exchange Traded options are:

1. *The counter-party risk:* On the exchange those risks are covered by the clearing house;
2. *Settlement and clearing* have to be specified in the OTC agreement, while on the stock exchange the clearing house will do this; and
3. *The regulatory oversight* is different.

ISDA — International Swap and Derivatives Organization

- developed the ISDA Master Agreement and other documentation
- engages with policy-makers and legislators around the world
- ISDA has over 800 member institutions from 64 countries. These members include:
  - market participants corporations, investment managers, government and supranational entities, insurance companies, energy and commodities firms, and international and regional banks.
  - others: exchanges, clearing-houses and repositories, law firms, accounting firms and other service providers.
- ISDA's work in three key areas
  - reducing counterparty credit risk,
  - increasing transparency, and
  - improving the industry's operational infrastructure

1. Supposedly, the first option buyer in the world was the ancient Greek mathematician and philosopher **Thales of Miletus** (ca. 624 – ca. 546 BCE). On a certain occasion, it was predicted that the season's olive harvest would be larger than usual, and during the off-season he acquired the right to use a number of olive presses the following spring. When spring came and the olive harvest was larger than expected he exercised his options and then rented the presses out at much higher price than he paid for his "option."
2. **Tulipomania** (March 1637): On February 24, 1637, the self-regulating guild of Dutch florists, in a decision that was later ratified by the Dutch Parliament, announced that all futures contracts written after November 30, 1636 and before the re-opening of the cash market in the early Spring, were to be interpreted as option contracts.
3. In **London**, puts and "refusals" (calls) first became well-known trading instruments in the 1690s during the reign of William and Mary.
4. **Privileges** were options sold OTC in nineteenth century America, with both puts and calls on shares offered by specialized dealers. Their exercise price was fixed at a rounded-off market price on the day or week that the option was bought, and the expiry date was generally three months after purchase. They were not traded in secondary markets.
5. In real **estate market**, call options have long been used to assemble large parcels of land from separate owners; e.g., a developer pays for the right to buy several adjacent plots, but is not obligated to buy these plots and might not unless he can buy all the plots in the entire parcel.

6. **Film or theatrical producers** often buy the right – but not the obligation – to dramatize a specific book or script.
7. **Lines of credit** give the potential borrower the right – but not the obligation – to borrow within a specified time period and up to a certain amount.
8. Many choices, or embedded options, have traditionally been included in **bond contracts**. For example, many bonds are convertible into common stock at the buyer's discretion, or may be called (bought back) at specified prices at the issuer's option.
9. Mortgage borrowers have long had the option to repay the loan early, which corresponds to a callable bond option.

- an **option** is the right to buy or sell at a pre-agreed price and moment
- a **future** is the obligation to sell or by at pre-agreed price and moment
- a **swap** is agreeing now to exchange a future cash flow with another

For example, one can swap an option where the buyer pays for example LIBOR +5bps during the life time of the option in stead of paying up front the option price . . . one can buy a future or an option on that structure . . .

```
# Let us plot the value of the call in function of the strike
FS <- seq(80, 120, length.out=150) # future spot price
X  <- 100                          # strike
P  <- 5                            # option premium
T  <- 3                            # time to maturity
r  <- 0.03                         # discount rate
payoff <- mapply(max, FS-X, 0)
profit <- payoff - P * (1 + r)^T

# Plot the results:
plot(FS, payoff,
     col='red', lwd = 3, type='l',
     main='LONG CALL value at maturity',
     xlab='Future strike price',
     ylab='$',
     ylim=c(-10,20)
     )
lines(FS, profit,
      col='blue', lwd=2)
text(105,8, 'Payoff', col='red')
text(115,5, 'Profit', col='blue')
```

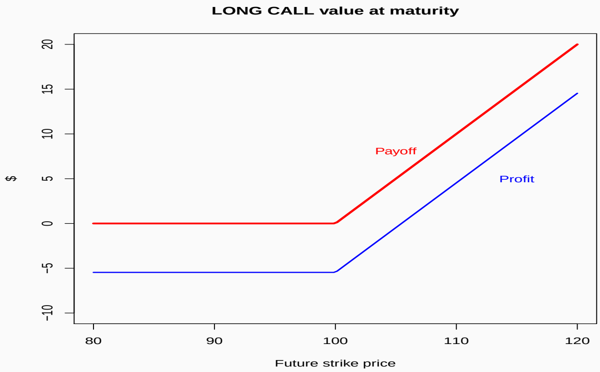


Figure 4: The intrinsic value of a long call illustrated with its payoff and profit. The profit is lower, since it takes into account that the option buyer has paid a fixed premium for the option.

```r
FS <- seq(80, 120, length.out=150) # future spot price
X  <- 100                          # strike
P  <- 5                            # option premium
T  <- 3                            # time to maturity
r  <- 0.03                         # discount rate
payoff <- - mapply(max, FS-X, 0)
profit <- P * (1 + r)^T + payoff

# Plot the results:
plot(FS, payoff,
     col='red', lwd = 3, type='l',
     main='SHORT CALL value at maturity',
     xlab='Future spot price',
     ylab='$',
     ylim=c(-20,10)
     )
lines(FS, profit,
      col='blue', lwd=2)
text(90,1.5, 'Payoff', col='red')
text(90,7, 'Profit', col='blue')
```

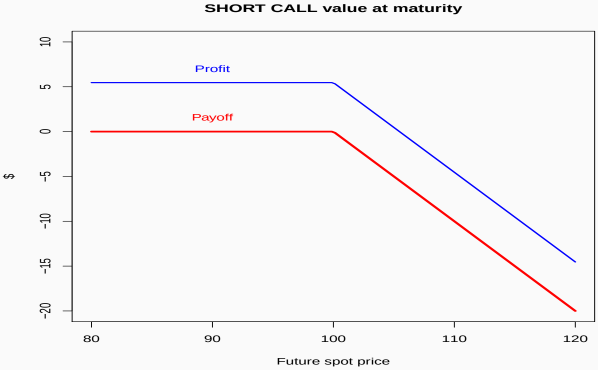


Figure 5: The intrinsic value of a short call illustrated with its payoff and profit. The profit is higher, since this the position of the option-wirter, so this party has got the premium at the start of the contract. Note that the loss is unlimited.

Long and Short Put at Maturity i

Long and Short Put at Maturity ii

```
FS <- seq(80, 120, length.out=150) # future spot price
X  <- 100                          # strike
P  <- 5                            # option premium
T  <- 3                            # time to maturity
r  <- 0.03                         # discount rate

# the long put:
payoff <- mapply(max, X - FS, 0)
profit <- payoff - P * (1 + r)^T

par(mfrow=c(1,2))

plot(FS, payoff,
     col='red', lwd = 3, type='l',
     main='LONG PUT at maturity',
     xlab='Future spot price',
     ylab='$',
     ylim=c(-20,20)
     )
lines(FS, profit,
      col='blue', lwd=2)
text(110,1,  'Payoff', col='red')
text(110,-4, 'Profit', col='blue')

# the short put:
payoff <- - mapply(max, X - FS, 0)
profit <- payoff + P * (1 + r)^T

plot(FS, payoff
```

Consider Put and a Call that

- are both European,
- have the same strike,
- have the same maturity, and
- have the same underlying.

$$C - P = D(F - X)$$

with

- C = the price of the Call
- P = the price of the Put
- F = the future price of the underlying
- X = the strike price
- D = the discount factor so that $S = D \times F$.

This can be rewritten as

$$C - P = S - D \times X$$

the right hand side is the same as buying a forward contract on the underlying with the strike as delivery price. So, a portfolio that is long a call and short a put is the same as being long a forward.

The Put-Call Parity

The Put-Call Parity can be rewritten as

$$C + D \times X = P + S$$

The option price is called "the premium."

Question #6

But how to calculate its value?

Use the basic statistical toolbox to calculate the expected value of the option. This involves:

1. making some (strong!) simplifying assumptions,
2. writing the payoff formulate of the European call,
3. calculating the expected value of a European call,
4. deriving the value of the European Put via the put-call-parity.

Assumptions for the BS model

1. Log-returns follow a Gaussian distribution on each time interval
2. The returns of one period are statistically independent of the return in other periods
3. Volatility and expected return exist and are and stable
4. The continuous time assumption:
   - interest rates are continuous: so that $e^{rt} = (1+i)^t$, implying that the compounded continuous rate can be calculated from the annual compound interest rate: $r = \log(1+i)$
   - also returns can be split infinitesimally and be expressed as a continuous rate.

The Black and Scholes Formula

- Call Price: $C(S, X, \tau, r, \sigma) = N(d_1)S - N(d_2)Xe^{-r\tau}$
- Put Price: $P(S, X, \tau, r, \sigma) = Xe^{-r\tau} - S + C(S, X, \tau, r, \sigma)$
- with:

$N(\cdot)$ the cumulative distribution function of the standard normal distribution
τ the time to maturity
S the spot price of the underlying asset
X the strike price
r the risk free rate (annual rate, expressed in terms of continuous compounding)
σ the volatility of the returns of the underlying asset

$$d_1 := \frac{\log\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)(\tau)}{\sigma\sqrt{\tau}}$$

$$d_2 := \frac{\log\left(\frac{S}{X}\right) + \left(r - \frac{\sigma^2}{2}\right)(\tau)}{\sigma\sqrt{\tau}} = d_1 - \sigma\sqrt{\tau}$$

To Fix the Ideas

We will work with the following example in mind (unless stated otherwise).

- $S = 100$
- $X = 100$ (when $S = X$ one says that the “option is at-the-money”)
- $\sigma = 20\%$
- $r = 2\%$
- $\tau = 1$ year

```r
# call_intrinsicVal
# Calculates the intrinsic value for a call option
# Arguments:
#    Spot   -- numeric -- spot price
#    Strike -- numeric -- the strike price of the option
# Returns
#    numeric -- intrinsic value of the call option.
call_intrinsicVal <- function(Spot, Strike) {max(Spot - Strike, 0)}
```

```r
# put_intrinsicVal
# Calculates the intrinsic value for a put option
# Arguments:
#    Spot   -- numeric -- spot price
#    Strike -- numeric -- the strike price of the option
# Returns
#    numeric -- intrinsic value of the put option.
put_intrinsicVal <- function(Spot, Strike) {max(-Spot + Strike, 0)}
```

```
# call_price
# The B&S price of a call option before maturity
# Arguments:
#    Spot   -- numeric -- spot price in $ or %
#    Strike -- numeric -- the strike price of the option  in $ or %
#    T      -- numeric -- time to maturity in years
#    r      -- numeric -- interest rates (e.g. 0.02 = 2%)
#    vol    -- numeric -- standard deviation of underlying in $ or %
# Returns
#    numeric -- value of the call option in $ or %
#
call_price <- function (Spot, Strike, T, r, vol)
 {
  d1 <- (log(Spot / Strike) + (r + vol ^ 2/2) * T) / (vol * sqrt(T))
  d2 <- (log(Spot / Strike) + (r - vol ^ 2/2) * T) / (vol * sqrt(T))
  pnorm(d1) * Spot - pnorm(d2) * Strike * exp(-r * T)
  }
```

```
# put_price
# The B&S price of a put option before maturity
# Arguments:
#    Spot   -- numeric -- spot price in $ or %
#    Strike -- numeric -- the strike price of the option  in $ or %
#    T      -- numeric -- time to maturity in years
#    r      -- numeric -- interest rates (e.g. 0.02 = 2%)
#    vol    -- numeric -- standard deviation of underlying in $ or %
# Returns
#    numeric -- value of the put option in $ or %
```

```r
# Examples:
call_price (Spot = 100, Strike = 100, T = 1, r = 0.02, vol = 0.2)
## [1] 8.916037

put_price  (Spot = 100, Strike = 100, T = 1, r = 0.02, vol = 0.2)
## [1] 6.935905
```

```r
# Long call
spot <- seq(50,150, length.out=150)
intrinsic_value_call <- apply(as.data.frame(spot),
                              MARGIN=1,
                              FUN=call_intrinsicVal,
                              Strike=100)
market_value_call    <- call_price(Spot = spot, Strike = 100,
                                   T = 3, r = 0.03, vol = 0.2)

# Plot the results:
plot(spot, market_value_call,
     type = 'l', col= 'red', lwd = 4,
     main = 'European Call option',
     xlab = 'Spot price',
     ylab = 'Option value')
text(115, 40, 'Market value', col='red')
lines(spot, intrinsic_value_call,
      col= 'forestgreen', lwd = 4)
text(130,15, 'Intrinsic value', col='forestgreen')
```

Plot the Value in function of the Spot Price of the Underlying ii

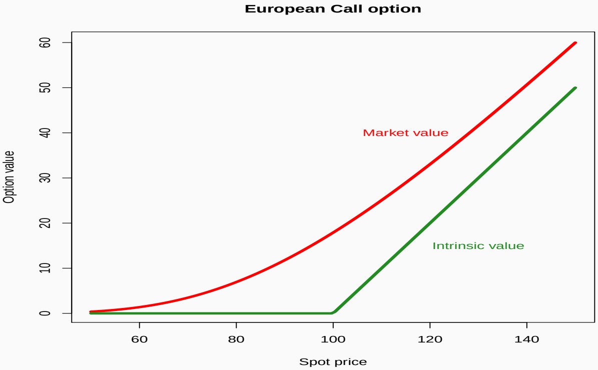


Figure 7: The price of a long call compared to its intrinsic value. The market value is always positive.

```
# Long put
spot <- seq(50,150, length.out=150)
intrinsic_value_put <- apply(as.data.frame(spot),
                             MARGIN=1,
                             FUN=put_intrinsicVal,
                             Strike=100)
market_value_put    <- put_price(Spot = spot, Strike = 100,
                                 T = 3, r = 0.03, vol = 0.2)

# Plot the results:
plot(spot, market_value_put,
     type = 'l', col= 'red', lwd = 4,
     main = 'European Put option',
     xlab = 'Spot price',
     ylab = 'Option value')
text(120, 8, 'market value', col='red')
lines(spot, intrinsic_value_put,
      col= 'forestgreen', lwd = 4)
text(75,10, 'intrinsic value', col='forestgreen')
```

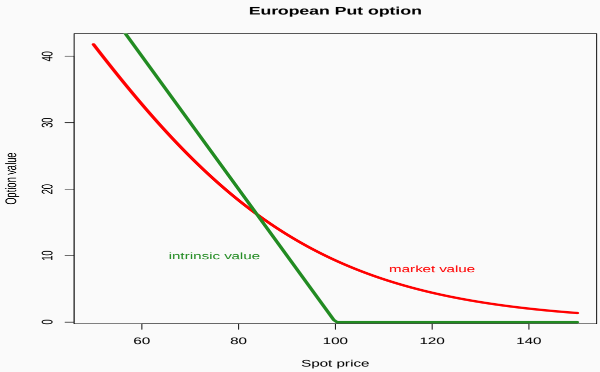


Figure 8: The price of a long put compared to its intrinsic value. Note that the market price of a put can be lower than its intrinsic value. This is because of the cost of carry.

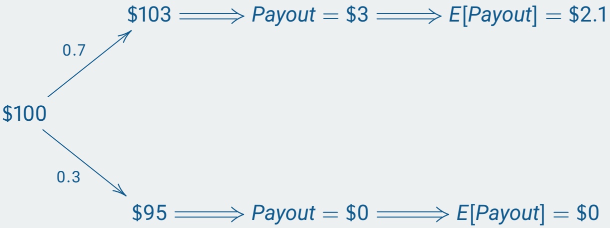
Strong Assumptions

- markets are efficient (follow a Wiener process, we can always buy and sell, etc.),
- the underlying is not paying any dividend (and if it is, this is a continuous flow),
- the log-returns at maturity are normally distributed (Gaussian distribution), and
- that the volatility is stable.

- What does this mean?
- A higher implied vol means a higher price for the option, so we're saying options far in-the-money or out-of-the-money cost more in real life than expected by the BS model.
- In the BS model, the log of the stock price was normally distributed at expiry, but more expensive options at distant strikes means that the real distribution has a higher-than-expected probability of ending up at extreme strikes, so the real probability has 'fat tails' relative to a normal distribution.

Example (One step binomial pricing model)

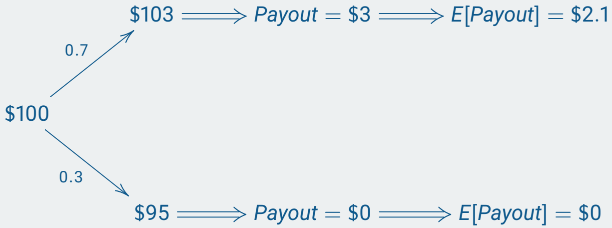
Calculate the price of a long ATM European call option; using one step in the binomial model and the following assumptions: $S_0 = \$100$, $p = 0.70$, $u = 1.03$ ($\equiv$ 3% increase), $d = 0.95$ ($\equiv$ 5% decrease), $r = 0.1$. Assume zero interest rates.
[0.5ex]1pt2mm



Binomial Model — Step 1

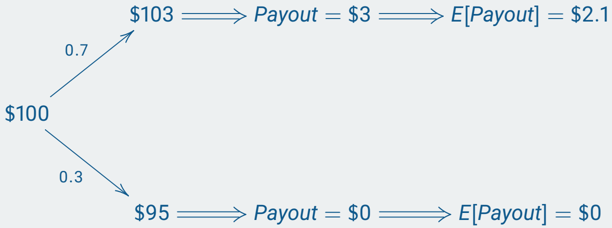
Example (One step binomial pricing model)

Calculate the price of a long ATM European call option; using one step in the binomial model and the following assumptions: $S_0 = \$100$, $p = 0.70$, $u = 1.03$ ($\equiv$ 3% increase), $d = 0.95$ ($\equiv$ 5% decrease), $r = 0.1$. Assume zero interest rates.
[0.5ex]1pt2mm



Example (One step binomial pricing model)

Calculate the price of a long ATM European call option; using one step in the binomial model and the following assumptions: $S_0 = \$100$, $p = 0.70$, $u = 1.03$ ($\equiv$ 3% increase), $d = 0.95$ ($\equiv$ 5% decrease), $r = 0.1$. Assume zero interest rates.
[0.5ex]1pt2mm

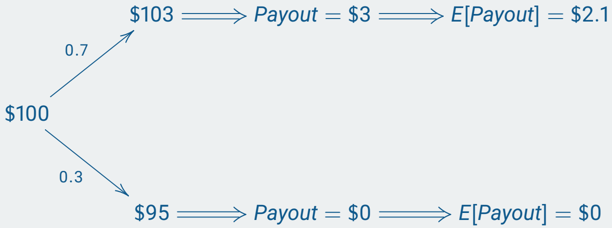


$\Rightarrow FV\left(P_{\text{equity}}\right) = 0.7 \times \$3 + 0.3 \times \$0 = 2.1$

Binomial Model — Step 1

Example (One step binomial pricing model)

Calculate the price of a long ATM European call option; using one step in the binomial model and the following assumptions: $S_0 = \$100$, $p = 0.70$, $u = 1.03$ ($\equiv$ 3% increase), $d = 0.95$ ($\equiv$ 5% decrease), $r = 0.1$. Assume zero interest rates.
[0.5ex]1pt2mm



$\Rightarrow FV\left(P_{\text{equity}}\right) = 0.7 \times \$3 + 0.3 \times \$0 = 2.1 \Rightarrow P_{\text{equity}} = \frac{\$2.1}{1+0.1} = \$1.91$

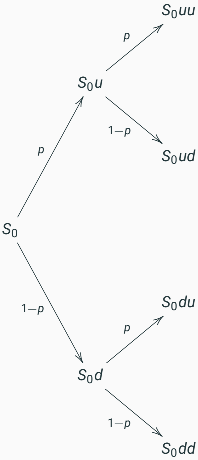


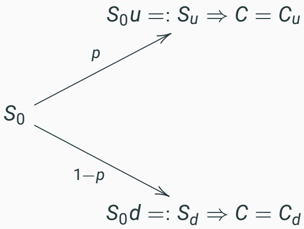
Figure 9: The first 2 steps of the binomial model.

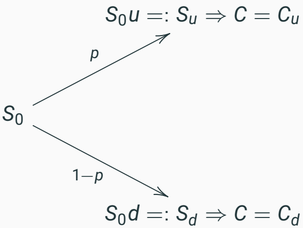
The Binomial Model: the idea

- try a limited number of possibilities (market scenarios)
- see what happens and try to find what can be "expected"

Binomial Model

1. Choose (u, d, p) consistent with some other theory of observation. For example, the Cox–Ross–Rubinstein model:
    - $u = e^{\sigma\sqrt{\delta t}}$
    - $d = e^{-\sigma\sqrt{\delta t}}$
    - $p = \frac{e^{R_{\mathrm{RF}}\delta t} - d}{u - d}$
2. Iterate till some convergence is satisfactory.
3. Discount the expected value of the option back to today.


$S_0u =: S_u \Rightarrow C = C_u$
p
S_0
$1-p$
$S_0d =: S_d \Rightarrow C = C_d$



The tree is built in the same way, but notice that:

- If two portfolios have the same payoff at time T, then they have the same price at time T-1.
- Choose the Equivalent Portfolio as "invest delta dollar in the underlying and borrow *B* cash," then

$$C = \delta S - B$$

If both portfolios have the same price now, then

$$C = \delta S - B$$

If both portfolios have the same price at T + 1, then

$$\begin{cases} C_u & = \delta S_u - (1+r)B \\ C_d & = \delta S_d - (1+r)B \end{cases}$$

We have now two equations with two unknown (δ and B), and hence easily find that:

$$\begin{cases} \delta & = \frac{C_u - C_d}{S_u - S_d} \\ B & = \frac{\delta S_u - C_u}{1+r} = \frac{\delta S_d - C_d}{1+r} \end{cases}$$

1. $\delta = \frac{C_u - C_d}{S_u - S_d}$
2. $B = \frac{\delta S_u - C_u}{1+r}$ or $B = \frac{\delta S_d - C_d}{1+r}$
3. $C = \delta S - B$

Example (– First order binomial model)

What is the value of a call option, assuming that: the strike price, X is \$100; the spot price, S is \$100, $S_d = \$98$, $S_u = \$105$, and the interest is 2%.

$\delta = \frac{5\$ - 0\$}{105\$ - 98\$} = 0.714 \Rightarrow B = \frac{0.714 \times 105\$ - 5\$}{1 + 0.02} = 68.60\$ \Rightarrow C = 0.714 \times 100\$ - 68.60\$ = \$2.88$

Risk Neutral vs the Equivalent Portfolio Model

- Both yield mathematically the same option value
- Risk neutral method
  - Is easier to calculate
  - Does not use the economic probability of the stock going up or down
- Equivalent portfolio method
  - Is more challenging computationally
  - Draws upon sound economic principle of arbitrage
  - Provides insight in option delta

Dependency on ...	Call	Put
Spot (value of underlying)	+	-
Volatility of underlying	+	+
Time to maturity	+	+
Interest rate	+	-
Dividend	-	+

Table 1: An overview of the price dependency for call and put options. A plus sign indicates that if the variable goes up, then the option premium goes up. The minus sign indicates that in the same case the option premium goes down.

	What	Call	Put
delta	$\frac{\partial C}{\partial S}$	$N(d_1)$	$-N(-d_1) = N(d_1) - 1$
gamma	$\frac{\partial^2 C}{\partial S^2}$	$\frac{N'(d_1)}{S\sigma\sqrt{\tau}}$	
vega	$\frac{\partial C}{\partial \sigma}$	$SN'(d_1)\sqrt{\tau}$	
theta	$\frac{\partial C}{\partial t}$	$-\frac{SN'(d_1)\sigma}{2\sqrt{\tau}} - rKe^{-r(\tau)}N(d_2)$	$-\frac{SN'(d_1)\sigma}{2\sqrt{\tau}} + rKe^{-r(\tau)}N(-d_2)$
rho	$\frac{\partial C}{\partial r}$	$K(\tau)e^{-r(\tau)}N(d_2)$	$-K(\tau)e^{-r(\tau)}N(-d_2)$

Table 2: An overview of “the Greeks:” the most relevant derivatives of the option price.

How can the option writer Insure him/herself?

- Buy or sell as much of the underlying as indicated by the delta.
- Repeat this step as often as possible.

Note that the delta of a call is positive and that of a put is negative.

Assume an asset with *strike* $= \$100$, *time to maturity* $= 5$ *years* , $\sigma = 0.2$, $r = 0.02$ (as continuous, so as percent: 1.98%).

tm 2 mat.	Spot	Delta	To hedge	Portf.	To buy	Cash
5.00	100.00	0.67	67.26	0.00	67.26	67.26
4.00	110.00	0.74	81.22	73.99	7.23	74.49
3.00	95.00	0.58	54.97	70.14	-15.18	59.32
2.00	110.00	0.73	80.55	63.65	16.91	76.22
1.00	105.00	0.67	70.50	76.89	-6.39	69.83
0.00	115.00	1.00	115.00	77.22	37.78	107.62

Table 3: Delta hedging of a hypothetical example where we only hedge our position once per year.

In previous example – where we adapt our position only once a year – the option writer has to buy and sell during the hedge. Note that:

- the payoff of the option is \$15, that is what the option writer pays the option buyer;
- our option writer has spent \$107 (non-discounted) and can sell this portfolio for \$115 (difference is [non-discounted] \$8), this is \$7 short for paying his customer, but he did get the premium;
- compared to the option price: 22.02% $\Longrightarrow$ \$15.02 profit (non-discounted);
- The difference at the end is \$37.78 (additional shares to buy), this is a big risk and results from leaving the position open for one year.

Covered Call ii

```r
## --- covered call ----

nbrObs     <- 100
the.S      <- 100
the.Strike <- 100
the.r      <- log (1 + 0.03)
the.T      <- 1
the.vol    <- 0.2
Spot.min   = 80
Spot.max   = 120
LegendPos  = c(.5,0.2)
Spot   <- seq(Spot.min,Spot.max,len=nbrObs)
val.end.call <-  - sapply(Spot, call_intrinsicVal, Strike = 100)
call.value <- call_price(the.S, the.Strike, the.T, the.r, the.vol)

d.underlying <- data.frame(Spot, Spot - 100,  'Underlying',   1)
d.shortcall  <- data.frame(Spot, val.end.call,  'Short call', 1)
d.portfolio  <- data.frame(Spot,
                           Spot + val.end.call + call.value - 100,
                           'portfolio', 1.1)
colnames(d.underlying) <- c('Spot', 'value', 'Legend','size')
colnames(d.shortcall)  <- c('Spot', 'value', 'Legend','size')
colnames(d.portfolio)  <- c('Spot', 'value', 'Legend','size')
dd <- rbind(d.underlying, d.shortcall, d.portfolio)
p <- qplot(Spot, value, data = dd, color = Legend, geom = "line",
           size=size )
p <- p + xlab('Value of the underlying' ) + ylab('Profit at maturity')
p <- p + theme(legend.position = LegendPos)
p <- p + scale_size(guide = 'none')
```

```r
## --- married put ----

LegendPos = c(.8,0.2)
Spot        <- seq(Spot.min,Spot.max,len=nbrObs)
val.end.put <-  sapply(Spot, put_intrinsicVal, Strike = 100)
put.value   <- - put_price(the.S, the.Strike, the.T, the.r, the.vol)

d.underlying <- data.frame(Spot, Spot - 100,  'Underlying', 1)
d.shortput   <- data.frame(Spot, val.end.put,  'Long put',  1)
d.portfolio  <- data.frame(Spot,
                           Spot + val.end.put + put.value - 100,
                           'portfolio',
                           1.1)
colnames(d.underlying) <- c('Spot', 'value', 'Legend','size')
colnames(d.shortput)   <- c('Spot', 'value', 'Legend','size')
colnames(d.portfolio)  <- c('Spot', 'value', 'Legend','size')
dd <- rbind(d.underlying,d.shortput,d.portfolio)
p  <- qplot(Spot, value, data = dd, color = Legend, geom = "line",
            size = size )
p <- p + xlab('Value of the underlying' ) + ylab('Profit at maturity')
p <- p + theme(legend.position = LegendPos)
p <- p + scale_size(guide = 'none')
print(p)
```

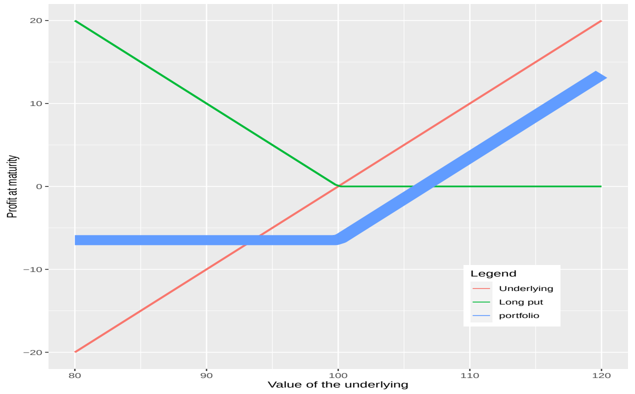


Figure 11: A married put is a put option combined with the underlying asset.

Collar i

Collar ii

```r
## --- collar ----

# Using the same default values as for the previous code block:
LegendPos = c(.6,0.25)
Spot          <-   seq(Spot.min,Spot.max,len=nbrObs)
val.end.call <- - sapply(Spot, call_intrinsicVal, Strike = 110)
val.end.put  <- + sapply(Spot, put_intrinsicVal, Strike = 95)
call.value   <- call_price(the.S, the.Strike, the.T, the.r, the.vol)
put.value    <- put_price(the.S, the.Strike, the.T, the.r, the.vol)

d.underlying <- data.frame(Spot, Spot - 100,   'Underlying', 1)
d.shortcall  <- data.frame(Spot, val.end.call, 'Short call', 1)
d.longput    <- data.frame(Spot, val.end.put,  'Long call',  1)
d.portfolio  <- data.frame(Spot, Spot + val.end.call + call.value +
                     val.end.put - put.value - 100, 'portfolio',1.1)
colnames(d.underlying) <- c('Spot', 'value', 'Legend','size')
colnames(d.shortcall)  <- c('Spot', 'value', 'Legend','size')
colnames(d.longput)    <- c('Spot', 'value', 'Legend','size')
colnames(d.portfolio)  <- c('Spot', 'value', 'Legend','size')
dd <- rbind(d.underlying,d.shortcall,d.longput,d.portfolio)
p  <- qplot(Spot, value, data=dd, color = Legend, geom = "line",
            size=size )
p <- p + xlab('Value of the underlying' ) + ylab('Profit at maturity')
p <- p + theme(legend.position = LegendPos)
p <- p + scale_size(guide = 'none')
print(p)
```

- *Knock-in:* This option only becomes active when a certain level (up or down) is reached.
- *Knock-out:* This option will have a fixed (zero or more) return when a certain level (up or down is reached); if the level is not reached it remains an option.
- *Barrier Option:* Another word for Knock-in or Knock-Out options.
- Options on “stock baskets” as opposed to options on stocks or indices, eventually accompanied by algorithms that chagne the basket during the lifetime of the option.

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- *Asian options:* The strike or spot is determined by the average price of the underlying taken at different moments.
- *Full Asianing:* The average of the Asian feature is taken over some moments over the whole lifetime of the option.
- *Look-back options:* The spot is determined as the best price of different moments
- *Russian Look-back options:* The look-back feature is unlimited (the best price over the whole lifetime of the option.
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Exotic Options

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- *Himalayan:* Payoff based on the performance of the best asset in the portfolio.
- *Everest:* Payoff based on the worst-performing securities in the basket.
- *Annapurna:* In which the option holder is rewarded if all securities in the basket never fall below a certain price during the relevant time period.
- *Atlas:* In which the best and worst-performing securities are removed from the basket prior to execution of the option.
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- *Bermuda Option:* An option where the buyer has the right to exercise at a set (always discretely spaced) number of times, so this structure is between an American and a European option.
- *Canary Option:* Can be exercised at quarterly dates, but not before a set time period has elapsed
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Question #7

Assume an initial portfolio of $100. Is it possible to construct a portfolio that

- offers capital protection in five years (ie. after five years, the price of the structure should be again $100)
- if the S&P500 increases our customer gets the same increase, e.g. if the S&P500 increases by 20% he gets 20% return
- if the S&P500 decreases, our customer gets $100 (so our customer never looses money on the investment)

A simple idea: combine:

- an option (or any combination thereof)
- a fixed term deposit

Example (capital protected structure)

Using our standard example parameters (see Slide 173), and assuming that

1. we also can place a deposit at $r = 2\%$,
2. that we build a five year capital protected structure,
3. that we have no transaction, nor spreads, nor holding costs, and
4. that our nominal amount is €$1,000$;

then we need to invest €1000 $\frac{1}{(1+0.02)^5} =$ €905.73 in the fixed term deposit in order to make it increase to 1000 in five year.
This leaves us €94.27 for buying an option. A call option on 5 years costs us €95.35 on a nominal of €$1,000$, so we can make a structure with a gearing of ca. 98.9%.

```r
##--------- the example of the capital protected structure
N           <- 5
nominal     <- 1000
inDeposit   <- nominal * (1.02)^(-N)
cst         <- 0.01               # in PERCENT
pvCosts     <- N * cst * nominal  # one should rather use the present value here
rest4option <- 1000 - inDeposit - pvCosts
callPrice   <- call_price (100, Strike = 100, T = 5, r = 0.02, vol = 0.02)
# reformulate this price as a percentage and then adjust to nominal
callPrice   <- callPrice / 100 * 1000
gearing     <- rest4option / callPrice
paste('The gearing is:', round(gearing * 100, 2))
## [1] "The gearing is: 46.43"
```

So, when adding 1% of annual costs to this structure (and not investing the provisions for these costs), our gearing decreases to 46.43%. That is less than half of the original gearing.